MTH 277, Quiz #15, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C zydx + xzdy + xydz$ on the smooth curve from $(1,2,3)$ *to* $(3,3,4)$.

$\varphi = xyz$ (conservative)

$3\cdot 3\cdot 4 - 1\cdot 2\cdot 3 =$

$36 - 6 = \boxed{30}$

2. Use Green's Theorem to evaluate $\int_C \underset{M}{(x^2 - y^2)}dx + \underset{N}{2xy}dy$ for $C: r = 1 + \cos\theta$.

this field is conservative
$\Rightarrow$ closed line integral $= 0$

$\frac{\partial N}{\partial x} = 2y \quad \frac{\partial M}{\partial y} = -2y$

$\iint_R 2y - (-2y)\, dA = \iint_R 4y\, dA = \int_0^{2\pi}\int_0^{1+\cos\theta} 4r\sin\theta\, r\, dr\, d\theta =$

$\int_0^{2\pi} \frac{4}{3}\sin\theta\, r^3\Big|_0^{1+\cos\theta} d\theta = \int_0^{2\pi}\frac{4}{3}\sin\theta(1+\cos\theta)^3\, d\theta =$

$u = \cos\theta$
$du = -\sin\theta\, d\theta$

$\int_0^{2\pi}\frac{4}{3}\sin\theta(1 + 3\cos\theta + 3\cos^2\theta + \cos^3\theta)\, d\theta =$

$\frac{4}{3}\int_0^{2\pi} \sin\theta + 3\sin\theta\cos\theta + 3\sin\theta\cos^2\theta + \sin\theta\cos^3\theta\, d\theta =$

$\frac{4}{3}\left[\cos\theta - \frac{3}{2}\cos^2\theta - \frac{3}{3}\cos^3\theta + \frac{1}{4}\cos^4\theta\right]_0^{2\pi} = \frac{4}{3}\left[(1-1) - \left(\frac{3}{2}\right)(1-1) - \frac{3}{3}(1-1) + \frac{1}{4}(1-1)\right]$

$= 0$