MTH 277, Quiz #17, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use the Divergence Theorem to evaluate $\int_S \int \vec{F}\cdot\vec{N}dS$ for $\vec{F}(x,y,z) = (xy^2+\cos z)\hat{\imath} + (x^2y+\sin z)\hat{\jmath} + e^z\hat{k}$ where $S: z = \frac{1}{2}\sqrt{x^2+y^2}, z = 8$.

$$8 = \frac{1}{2}\sqrt{x^2+y^2}$$
$$16 = \sqrt{x^2+y^2}$$
$$256 = x^2+y^2 \qquad r = 16$$

$$\vec{\nabla}\cdot\vec{F} = y^2+x^2 = r^2$$

$$\iint_S F.N\,dS = \iiint_V \vec{\nabla}\cdot\vec{F}\,dV = \int_0^{2\pi}\int_0^{16}\int_{\frac{1}{2}r}^{8} r^2\,r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{16} r^3 z\Big|_{\frac{1}{2}r}^{8} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{16} 8r^3 - \frac{1}{2}r^4\,dr\,d\theta = \int_0^{2\pi} 2r^4 - \frac{1}{10}r^5\Big|_0^{16} = \int_0^{2\pi} 131{,}072 - 104{,}857.6\,d\theta$$

$$= 2\pi(26214.4) = 52{,}428.8\pi$$

2. Use Stokes' Theorem to evaluate $\int_C \vec{F}\cdot d\vec{r}$ for $\vec{F} = 4xz\hat{\imath} + y\hat{\jmath} + 4xy\hat{k}$ for the surface $S: z = 9 - x^2 - y^2, z \geq 0$.

$$\nabla\times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4xz & y & 4xy \end{vmatrix} = (4x-0)\hat{\imath} - (4y-4x)\hat{\jmath} + (0-0)\hat{k}$$
$$4x\hat{\imath} + (4x-4y)\hat{\jmath} = \langle 4x, 4x-4y, 0\rangle$$

$$G = z - 9 + x^2 + y^2$$
$$\nabla G = \langle 2x, 2y, 1\rangle$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$(\nabla\times F)\cdot\nabla G = 8x^2 + 8xy - 8y^2$$

$$\int_0^{2\pi}\int_0^3 (8r^2\cos^2\theta + 8r^2\cos\theta\sin\theta - 8r^2\sin^2\theta)\,r\,dr\,d\theta =$$
$$\int_0^{2\pi}\int_0^3 8r^3\cos^2\theta + 8r^3\cos\theta\sin\theta - 8r^3\sin^2\theta\,dr\,d\theta = \int_0^{2\pi}\frac{8}{4}r^4(\cos^2\theta + \cos\theta\sin\theta - \sin^2\theta)\,d\theta\Big|_0^3$$

$$\int_0^{2\pi} 162(\cos 2\theta + \cos\theta\sin\theta)\,d\theta = 162\left[\frac{1}{2}\sin 2\theta + \frac{1}{2}\sin^2\theta\right|_0^{2\pi} = 0$$

if you use $\nabla G = \hat{k}$ for the circle in the plane, you get 0 faster