MTH 291, Quiz #3, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the differential equation $y' = x(1+y)$ using separation of variables.

$$\int \frac{dy}{1+y} = \int x\,dx \qquad y(x) = Ae^{\frac{1}{2}x^2} - 1$$

$$\ln(1+y) = \frac{1}{2}x^2 + C$$

$$1+y = Ae^{\frac{1}{2}x^2}$$

2. Solve the differential equation $xy + y' = 100x$ by the method of integrating factors (reverse product rule).

$$y' + xy = 100x \qquad \mu = e^{\int x\,dx} = e^{\frac{1}{2}x^2}$$

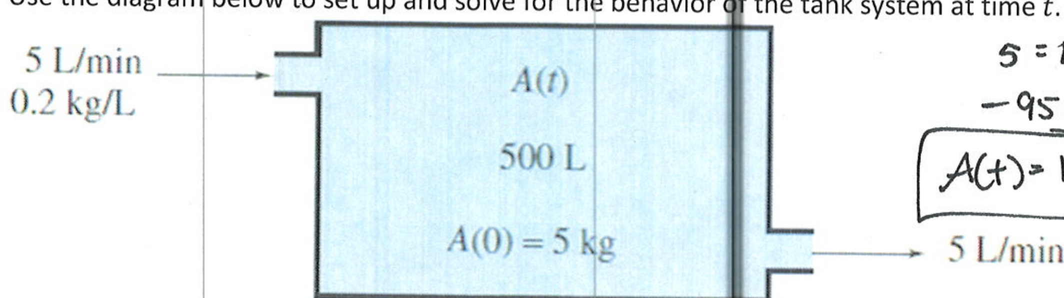
$$e^{\frac{1}{2}x^2}y' + xe^{\frac{1}{2}x^2}y = 100xe^{\frac{1}{2}x^2}$$

$$\int \left(e^{\frac{1}{2}x^2}y\right)' = \int 100xe^{\frac{1}{2}x^2}$$

$$e^{\frac{1}{2}x^2}y = 100e^{\frac{1}{2}x^2} + C$$

$$\boxed{y = 100 + Ce^{-\frac{1}{2}x^2}}$$

3. Use the diagram below to set up and solve for the behavior of the tank system at time t.



$$5 = 100 + A_0e^{-\frac{1}{100}0}$$

$$-95 = A_0$$

$$\boxed{A(t) = 100 - 95e^{-\frac{1}{100}t}}$$

$$\frac{dA}{dt} = 5 \cdot 0.2\,\frac{kg}{min} - \frac{A \cdot 5}{500}$$

$$\frac{dA}{dt} = 1 - \frac{A}{100} = \frac{-1}{100}(A - 100)$$

$$\frac{dA}{A-100} = -\frac{1}{100}\,dt$$

$$\ln(A-100) = -\frac{1}{100}t + C$$

$$A - 100 = A_0e^{-\frac{1}{100}t}$$

$$\boxed{A(t) = 100 + A_0e^{-\frac{1}{100}t}}$$