

Instructions: You must show all work to receive full credit for the problems below. You may **not** use a calculator for this section of the exam and all answers without work will receive minimal credit. Use exact answers.

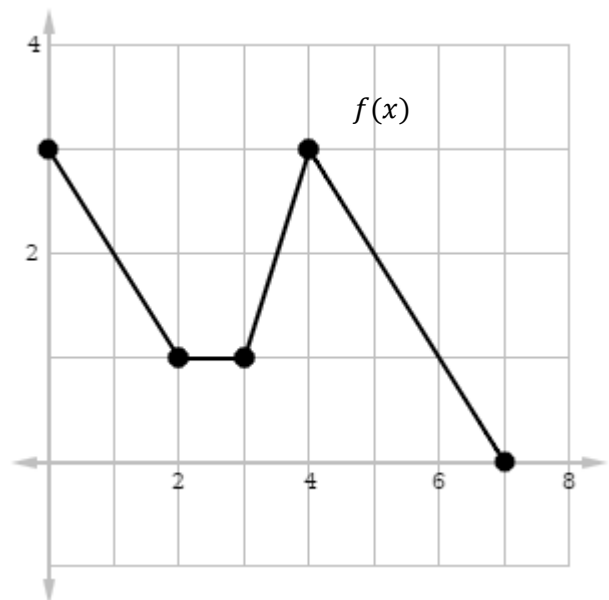
1. Find the absolute extrema of the function $f(x) = x^4 - 2x^2 + 5$ on the interval $[-2,2]$. (10 points)

2. Find the equation of the tangent line to the implicitly defined function $x^4 - x^2y^3 = 12$ at the point $(-2,1)$. (12 points)

3. Integrate $\int \frac{4}{\sqrt[3]{x}} + \frac{3}{4}e^{6x} - \frac{7}{x} dx$. (8 points)

4. Find $f(x)$ if $f'(x) = 8x^2 + 4x - 2$. Find the constant of integration if $f(0) = 6$. (10 points)

5. Use graph to the right to evaluate the integral $\int_0^7 f(x) dx$ geometrically. (12 points)



6. Find the error(s) in the following work. Identify each error, explain why it is incorrect, and make an appropriate correction. [Hint: there is at least one mistake.] (10 points)

$$\begin{aligned}\int_1^2 \ln x - e^x dx &= \left[\frac{1}{x} - e^x \right]_1^2 \\ &= \left(\frac{1}{2} - e^2 \right) - (1 - e^1) \\ &= e - e^2 - \frac{1}{2}\end{aligned}$$

7. Find the area under the curve $f(x) = x^3 - 3x$ over the interval $[-1,1]$. (10 points)

8. Find the area bounded by $f(x) = x$ and $g(x) = \sqrt[4]{x}$. Sketch the graph. (10 points)

9. Integrate. (10 points each)

a. $\int x^3 e^x dx$

b. $\int \ln x dx$

c. $\int \frac{e^{1/x}}{x^2} dx$

d. $\int \frac{1}{1+7x} dx$

11. Suppose that the price p in dollars and number of sales x of a certain item follow the equation $5p + 4x + 2px = 60$. Suppose also that p and x are both functions of time, measured in days. Find the rate at which x is changing $\left(\frac{dx}{dt}\right)$ when $x = 3, p = 5, \frac{dp}{dt} = 1.5$. (12 points)

12. Suppose that P_0 is invested in the Mandelbrot Bond Fund for which interest is compounded continuously at 5.9% per year. That is the balance P grows at the rate given by $\frac{dP}{dt} = 0.059P$.

a. Find the function that satisfies the equation in terms of P_0 and 0.059. (5 points)

b. Suppose that \$1000 is invested. What is the balance in the account after 1 year? (5 points)

c. What is the balance in the account after 2 years? (6 points)

d. When will an investment of \$1000 double itself? (6 points)

13. Carbon-14 has a decay rate that is modeled by the equation $\frac{dN}{dt} = -0.00012097N$, where t is in years. How old is an ivory tusk if 40% of its original Carbon-14 remains? (12 points)

14. The elasticity of demand is given by $E(x) = -\frac{x D'(x)}{D(x)}$. Find the elasticity for $D(x) = \sqrt{600 - x}$, at $x = 100$. (10 points)

15. Approximate the area under the curve $f(x) = \frac{1}{x^2}$ on the interval $[1,7]$ by computing the area under 6 rectangles (using the left-hand rule). (15 points)