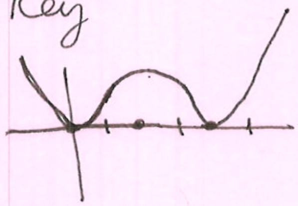
MTH 261 Homework #2 Key

1. a. 

b. 

2a. $f(x) = (2x+1)^2 = (4x^2+4x+1) \rightarrow f'(x) = 8x+4$

b. $f(x) = (2x+1)^2 = (2x+1)(2x+1) \rightarrow f'(x) = 2(2x+1) + 2(2x+1) =$
$4(2x+1) = 8x+4$

c. $f(x) = (2x+1)^2 \rightarrow f'(x) = 2(2x+1)^1(2) = 4(2x+1) = 8x+4$

d. they all produce the same results

3a. $G(x) = (8x+\sqrt{x})(5x^2+3)$
$G'(x) = \left(8 + \frac{1}{2\sqrt{x}}\right)(\ \ x^2+3) + (8x+\sqrt{x})(10x)$

b. $F(x) = (5x+2)^3(2x-3)^8$
$F'(x) = 3(5x+2)^2(5)(2x-3)^8 + 8(2x-3)^7(2)(5x+2)^3$

c. $H(x) = x^3(6x+1)^2(7x-2)^4$
$H'(x) = 3x^2(6x+1)^2(7x-2)^4 + x^3 \cdot 2(6x+1)(6)(7x-2)^4 + x^3(6x+1)^2 4(7x-2)^3(7)$

d. $g(x) = (4x^2+3x)e^{x^2-7x}$
$g'(x) = (8x+3)e^{x^2-7x} + (4x^2+3x)e^{x^2-7x}(2x-7)$

e. $y = \dfrac{\sqrt[3]{x}-7}{\sqrt{x}+3}$ $\quad y' = \dfrac{\frac{1}{3}x^{-2/3}(\sqrt{x}+3) - \frac{1}{2}x^{-1/2}(\sqrt[3]{x}-7)}{(\sqrt{x}+3)^2}$

f. $f(x) = \sqrt{\dfrac{x^2+x}{x^2-x}}$ $\quad f'(x) = \dfrac{1}{2}\sqrt{\dfrac{x^2-x}{x^2+x}}\left[\dfrac{(2x+1)(x^2-x) - (2x-1)(x^2+x)}{(x^2-x)^2}\right]$

g. $g(x) = (2x^3 + (4x-5)^2)^6$
$g'(x) = 6(2x^3+(4x-5)^2)^5[6x^2+2(4x-5)(4)]$

4. a. $y = -\frac{4}{5}e^{x^3}$ $\quad y' = -\frac{12}{5}x^2e^{x^3}$

b. $f(x) = \ln\left(\frac{x^2-7}{x}\right) = \ln(x^2-7) - \ln(x)$
$f'(x) = \dfrac{2x}{x^2-7} - \dfrac{1}{x}$

4c. $h(x) = \sqrt{x^2 + \sqrt{1-3x}}$ $\quad h'(x) = \frac{1}{2}\left(x^2 + \sqrt{1-3x}\right)^{-1/2}\left[2x + \frac{1}{2}(1-3x)^{-1/2}(-3)\right]$

d. $g(x) = \dfrac{e^{3x}}{x^6} = x^{-6}e^{3x}$ $\quad g'(x) = -6x^{-7}e^{3x} + 3x^{-6}e^{3x}$

e. $p(x) = (\ln x)^3 \ln\left[\ln\left(e^{x^2}+6\right)\right]$

$p'(x) = 3(\ln x)^2 \cdot \frac{1}{x} \ln\left[\ln\left(e^{x^2}+6\right)\right] + (\ln x)^3 \dfrac{1}{\ln\left(e^{x^2}+6\right)} \cdot \dfrac{1}{e^{x^2}+6} \cdot 2xe^{x^2}$

5. $\dfrac{d^3y}{dx^3}$, f''', $f^{(3)}$, $D_x^3 f(x)$

6. $f(x) = x^6 - x^3 - \frac{2}{x}$

$f'(x) = 6x^5 - 3x^2 + 2x^{-2}$

$f''(x) = 30x^4 - 6x^2 - 4x^{-3}$

7. $f(x) = \sqrt[4]{x} - \sqrt{x} = x^{1/4} - x^{1/2}$

$f'(x) = \frac{1}{4}x^{-3/4} - \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{3}{16}x^{-7/4} + \frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{21}{64}x^{-11/4} - \frac{3}{8}x^{-5/2}$

$f^{iv}(x) = -\frac{231}{256}x^{-15/4} + \frac{15}{16}x^{-7/2}$

$f^{v}(x) = \frac{3465}{1024}x^{-19/4} - \frac{105}{32}x^{-9/2}$