MTH 261 Homework #5 Key

1\. a. $\int_1^7 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^7 = -\frac{1}{7}+\frac{1}{1} = \frac{6}{7} \approx 0.8571$

b. $\int_0^5 x^2+1\,dx = \frac{1}{3}x^3+x\Big|_0^5 = \frac{125}{3}+5 = \frac{140}{3} \approx 46.67$

c. $\int_1^2 e^x+1\,dx = e^x+x\Big|_1^2 = e^2+2-e^1-1 = e^2-e+1 \approx 5.6708$

d. $\int_3^8 \ln(x^3-1)\,dx = 24.9710$

2\. $A_1 = \frac{1}{2}(2)(1.5) = 1.5$ $\quad A_2 = 1.5 * 4 = 3$ (see graph attached)

area $\int_0^6 f(x)\,dx = 1.5+3 = 4.5$

3\. a. $\int_0^1 x^{1/4}-x\,dx = \frac{4}{5}x^{5/4}-\frac{1}{2}x^2\Big|_0^1 = \frac{4}{5}-\frac{1}{2} = \frac{3}{10}$

b. $\int_1^3 x^2+1-x^2\,dx = \int_1^3 1\,dx = x\Big|_1^3 = 3-1 = 2$

c. $\int_0^3 3-x\,dx = 3x-\frac{1}{2}x^2\Big|_0^3 = 9-\frac{9}{2} = \frac{9}{2}$

d. $\int_{-2}^0 x+6-(-2x)\,dx + \int_0^2 x+6-x^3\,dx =$

$\int_{-2}^0 3x+6\,dx + \int_0^2 x+6-x^3\,dx = \left[\frac{3}{2}x^2+6x\Big|_{-2}^0 + \left[\frac{1}{2}x^2+6x-\frac{1}{4}x^4\Big|_0^2 = 6+10 = 16$

4a. $\int \frac{1}{1+t}\,dt = \ln|1+t|+C \qquad f(0)=1 \qquad \ln|1+t|+1$

$\ln|1+0|+C = 1 \Rightarrow C=1$

b. $\int xe^{x^2}\,dx = \frac{1}{2}e^{x^2}+C \qquad u=x^2,\ du=2x\,dx,\ \frac{1}{2}du = x\,dx \qquad \int \frac{1}{2}e^u\,du = \frac{1}{2}e^{x^2}+C$

c. $\int \frac{e^t}{3+e^t}\,dt \qquad u=3+e^t,\ du=e^t\,dt \qquad \int \frac{du}{u} = \ln u + C = \ln(3+e^t)+C$

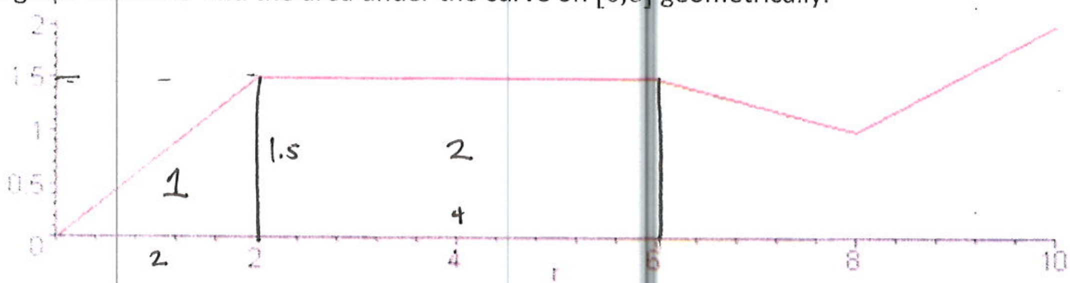
d. $\int \frac{4}{x\ln x}\,dx \qquad u=\ln x,\ du=\frac{1}{x}\,dx \qquad \to \int \frac{4}{u}\,du = 4\ln u + C \Rightarrow 4\ln(\ln x)+C$

MTH 261, Written Homework #5, Spring 2019 Name ______________________

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. Some problems will require the use of technology (such as Excel, your calculator, or free widgets online). Complete your work on a separate page and attach to this cover sheet.

1. Evaluate the definite integrals and compare the results obtained from the approximations on the last homework.
   a. $f(x) = \frac{1}{x^2}$. $[1,7]$
   b. $f(x) = x^2 + 1, [0,5]$
   c. $f(x) = e^x + 1, [1,2]$
   d. $f(x) = \ln(x^3 - 1), [3,8]$ (Do this integral numerically on your calculator or with a computer algebra system.)

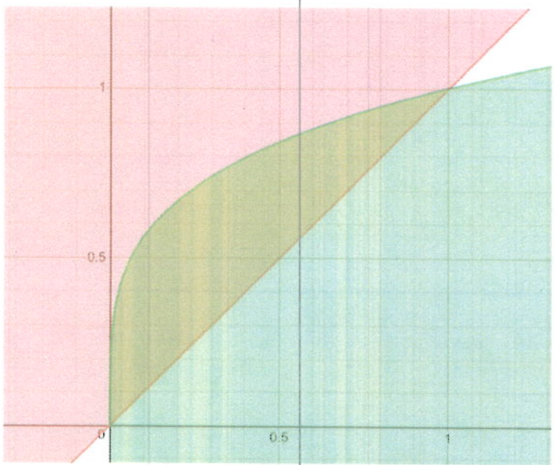
2. Use the graph below to find the area under the curve on $[0,6]$ geometrically.

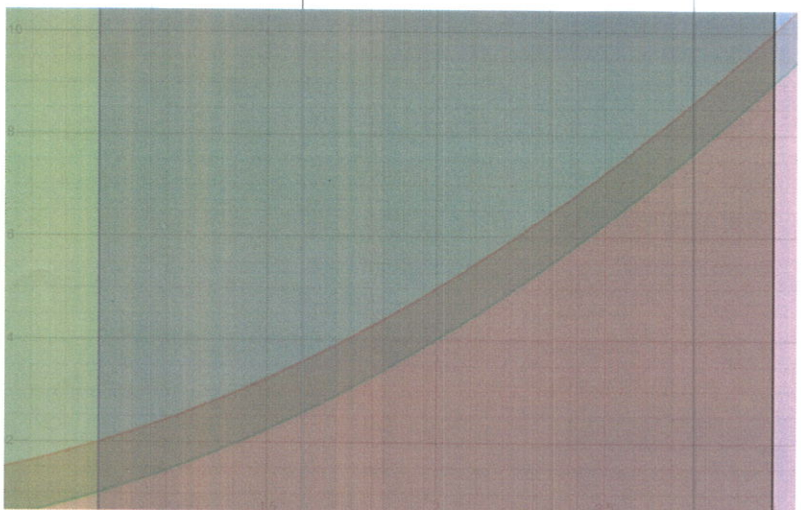


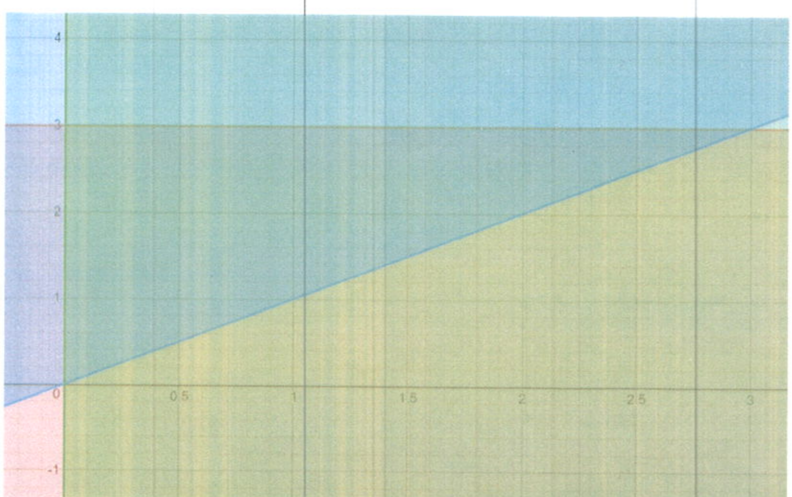
3. Sketch the set of curves and find the area bounded between them.
   a. $y = x, y = \sqrt[4]{x}$
   b. $y = x^2 + 1, y = x^2, x = 1, x = 3$
   c. $y = 3, y = x, x = 0$
   d. $y = x + 6, y = -2x, y = x^3$

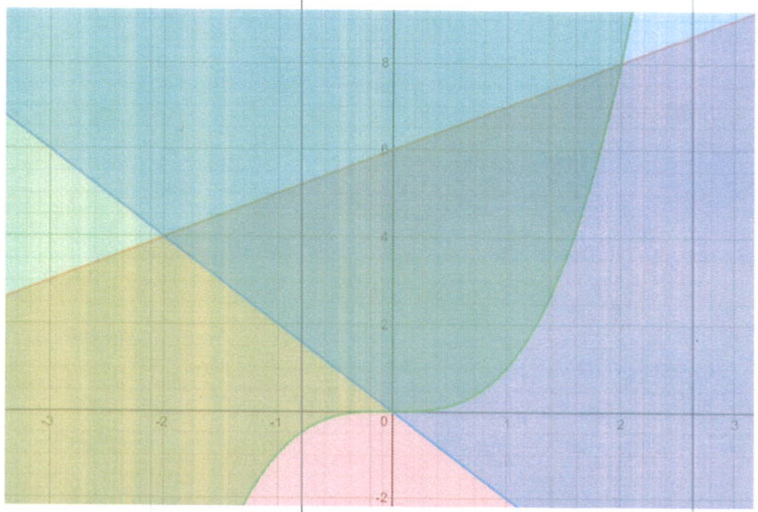
4. Integrate. If an initial value is given, find the constant of integration.
   a. $\int \frac{1}{1+t} dt, f(0) = 1$
   b. $\int xe^{x^2} dx$
   c. $\int \frac{e^t}{3+e^t} dt$
   d. $\int \frac{4}{x \ln x} dx$
   e. $\int x\sqrt{x+1} dx, f(0) = 4$

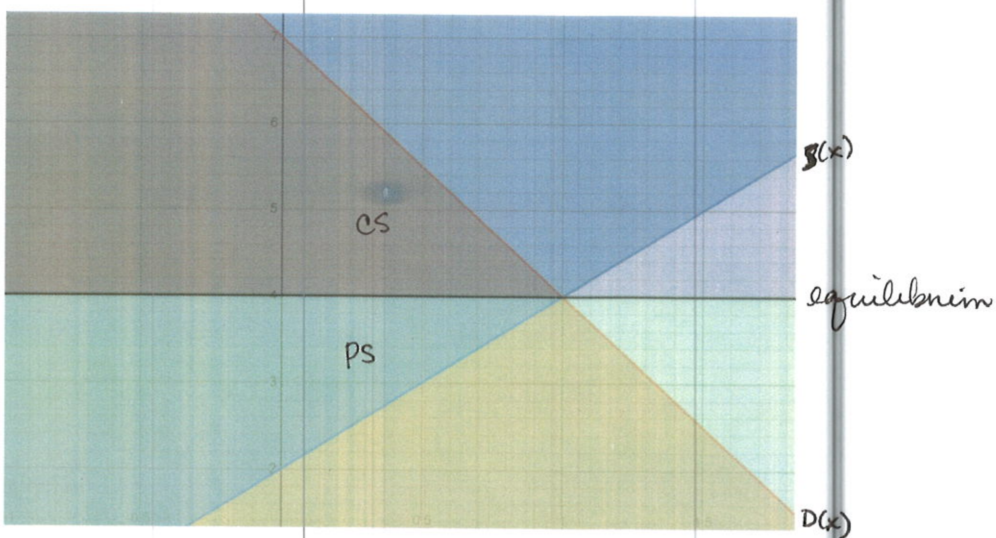
5. Sketch graphs of $D(x) = -3x + 7$ and $S(x) = 2x + 2$ (the demand and supply functions respectively). Find their equilibrium point, and use that to find the consumer's surplus and the producer's surplus. On the graph, illustrate these graphically.


1
0.5
0
0.5
1


10
8
6
4
2


4
3
2
1
0
0.5
1
1.5
2
2.5
3
-1




S(x)
CS
equilibrium
PS
D(x)

4e. $\int x\sqrt{x+1}\,dx \quad f(0)=4$

Method I:

$u=\sqrt{x+1} \quad u^2=x+1$

$2u\,du=dx \quad u^2-1=x$

$$\int (u^2-1)u\cdot 2u\,du = \int 2u^4-2u^2\,du = \frac{2}{5}u^5-\frac{2}{3}u^3+C$$

$$= \frac{2}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}+C$$

Method 2:

$u=x \quad dv=(x+1)^{1/2}dx$

$du=dx \quad v=\frac{2}{3}(x+1)^{3/2}$

$$\frac{2}{3}x(x+1)^{3/2}-\int \frac{2}{3}(x+1)^{3/2}dx$$

$$\frac{2}{3}x(x+1)^{3/2}-\frac{2}{3}\cdot\frac{2}{5}(x+1)^{5/2}+C$$

$$\frac{2}{3}x(x+1)^{3/2}-\frac{4}{15}(x+1)^{5/2}+C$$

these solutions are algebraically equivalent or differ by only a constant

5. $D(x)=-3x+7 \quad S(x)=2x+2 \quad (1,4)$

$-3x+7=2x+2 \quad 4=2(1)+2$

$-5x=-5$

$x=1$

$$CS=\int_0^1 -3x+7-4\,dx=\int_0^1 -3x+3\,dx=-\frac{3}{2}x^2+3x\Big|_0^1=-\frac{3}{2}+3=\frac{3}{2}$$

$$PS=\int_0^1 4-(2x+2)\,dx=\int_0^1 2-2x\,dx=2x-x^2\Big|_0^1=2-1=1$$