MTH 266, Quiz #3, Spring 2019 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. There are three "tests" for determining if a function $T(\vec{x}) = A\vec{x}$ is a linear transformation.
   i. $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$
   ii. $T(c\vec{u}) = cT(\vec{u})$
   iii. $T(\vec{0}) = \vec{0}$

a. For the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, show using generic vectors, that this is a linear transformation.

Let $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ i. $T(\vec{u}) = \begin{bmatrix} u_1 - u_2 \\ 2u_1 + 3u_2 \end{bmatrix}$ $T(\vec{v}) = \begin{bmatrix} v_1 - v_2 \\ 2v_1 + 3v_2 \end{bmatrix}$

$\vec{u}+\vec{v} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}$, $T(\vec{u}+\vec{v}) = \begin{bmatrix} u_1+v_1 - (u_2+v_2) \\ 2(u_1+v_1) + 3(u_2+v_2) \end{bmatrix} = \begin{bmatrix} u_1 - u_2 + v_1 - v_2 \\ 2u_1 + 3u_2 + 2v_1 + 3v_2 \end{bmatrix}$

$= T(\vec{u}) + T(\vec{v})$

ii. $c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$, $T(c\vec{u}) = \begin{bmatrix} cu_1 - cu_2 \\ 2cu_1 + 3cu_2 \end{bmatrix}$

$cT(\vec{u}) = c\begin{bmatrix} u_1 - u_2 \\ 2u_1 + 3u_2 \end{bmatrix} = \begin{bmatrix} cu_1 - cu_2 \\ 2cu_1 + 3cu_2 \end{bmatrix} = T(c\vec{u})$

iii. $T(\vec{0}) = \begin{bmatrix} 0 - 0 \\ 2(0) + 3(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{0}$ Thus, is it a linear transformation

b. Show that the derivative operator $T(f) = \frac{df}{dx}$ is a linear operator, using generic functions and properties of derivatives learned in Calculus I.

$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

$\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$

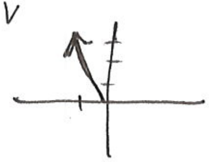
$\frac{d}{dx}[0] = 0$, all by properties of derivatives

thus, this operator satisfies the definition of a linear transformation.

2. Graph the vector $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$, then on a separate graph, plot the vector under the indicated linear transformation. Use that information to determine what kind of linear transformation it is: reflection (specify the axis), rotation, expansion or compression (specify direction), shear, other.

a. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad A\vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\vec{v}$



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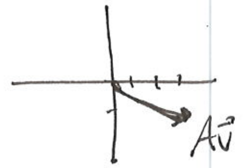
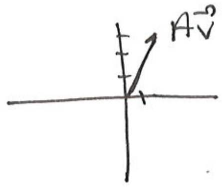


reflection over $x = y$

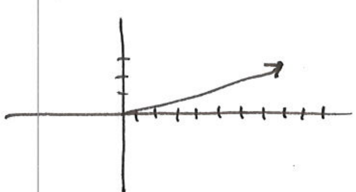
b. $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad Av = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$



reflection over y-axis

c. $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $\quad Av = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

horizontal skew



d. $A = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$ $\quad A\vec{v} = \begin{bmatrix} -2 \\ 3/2 \end{bmatrix}$

Scales horizontally by 2 (stretch)
Scales vertically by 1/2 (compression)