MTH 161, Exam #2, Spring 2021 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing"; partial credit will not be possible.

1. Simplify the following expressions. Write each in standard form. (6 points each)
   a. $7-(-9+2i)-(-17-i)$

      $+7+9-2i+17+i$

      $16-2i+17+i$

      $33-i$

   b. $(5-2i)^2$ $= 25 - 10i + 4i^2$

      $25-20i-4 = 21-20i$

   c. $\frac{2+3i}{2+i} \cdot \frac{2-i}{2-i} = \frac{4-2i+6i-3i^2}{4+1} = \frac{7+4i}{5} = \frac{7}{5}+\frac{4}{5}i$

      ($-3i^2 = +3$)

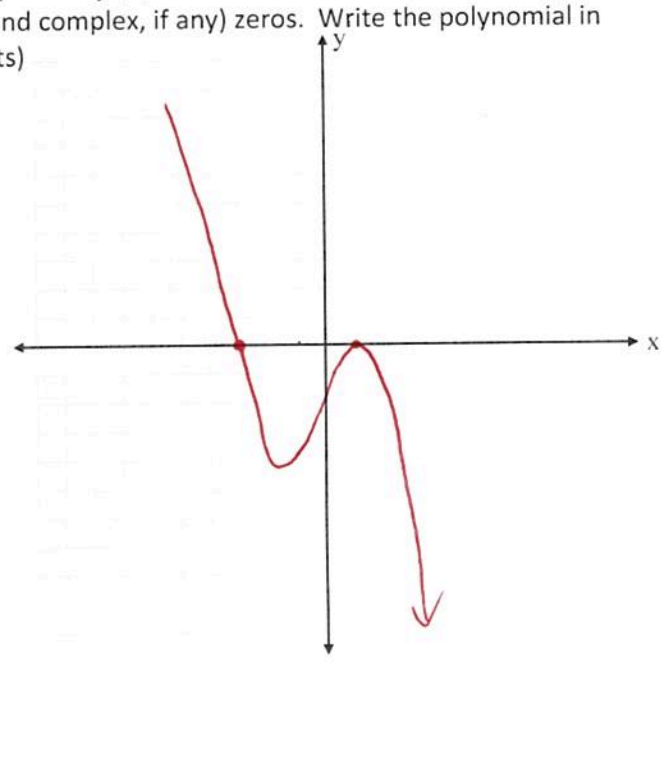
2. Find all the possible rational zeros of the polynomial $f(x)=-x^3-x^2+5x-3$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. Graph the function. (10 points)

$$
\begin{array}{r}
-x^2+2x-1 \\
x+3 \,\big)\, \overline{-x^3-x^2+5x-3} \\
+x^3+3x^2 \\
\hline
2x^2+5x-3 \\
-2x^2\mp 6x \\
\hline
-x-3 \\
+x+3 \\
\hline
0
\end{array}
$$

$-(x+3)(x^2-2x+1) =$

$-(x+3)(x-1)^2$

Zeros are $-3, 1$



3. Divide using polynomial long division. Write the solution as $Quotient + \frac{Remainder}{Divisor}$. (8 points each)

a. $\frac{2x^5-8x^4+2x^3+x^2}{2x^3+1}$

$$x^2-4x+1$$
$$2x^3+0x^2+0x+1 \overline{)\,2x^5-8x^4+2x^3+x^2+0x+0}$$
$$-2x^5+0x^4+0x\mp x^2$$
$$-8x^4+2x^3+0x^2+0x+0$$
$$+8x^4+0x^3+0x^2\mp 4x$$
$$2x^3+0x^2+4x+0$$
$$-2x^3+0x^2+0x\mp 1$$
$$4x-1$$

$$x^2-4x+1+\frac{4x-1}{2x^3+1}$$

b. $\frac{x^5+x^3-2}{x-1}$

$$x^4+x^3+2x^2+2x+2$$
$$x-1 \overline{)\,x^5+0x^4+x^3+0x^2+0x-2}$$
$$-x^5+x^4$$
$$x^4+x^3+0x^2+0x-2$$
$$-x^4+x^3$$
$$2x^3+0x^2+0x-2$$
$$-2x^3+2x^2$$
$$2x^2+0x-2$$
$$-2x^2+2x$$
$$2x-2$$
$$2x-2$$

$$x^4+x^3+2x^2+2x+2$$

4. Redo problem 3b: $\frac{x^5+x^3-2}{x-1}$ using Synthetic Division. Show that the answer you obtain is consistent with your answer in 3b.

$$\begin{array}{c|cccccc} 1 & 1 & 0 & 1 & 0 & 0 & -2 \\ & & 1 & 1 & 2 & 2 & 2 \\ \hline & 1 & 1 & 2 & 2 & 2 & \boxed{0} \end{array}$$

$$x^4+x^3+2x^2+2x+2$$

5. Write a polynomial with the given properties. You may leave the polynomial in factored form **with real coefficients**. (8 points each)

a. $n = 3$, zeros are $1, 5$ (multiplicity two) and $f(-1) = -104$

$a(x-1)(x-5)^2$

$a(-1-1)(-1-5)^2 = -104$

$a(-2)(36) = -104$

$a(-72) = -104$

$a = 13/9$

$f(x) = \frac{13}{9}(x-1)(x-5)^2$

b. $n = 4$, zeros $-2, 5, 3 + 2i$, and $f(1) = -96$

$(x+2)(x-5)(x-3-2i)(x-3+2i)$

$a(x+2)(x-5)(x^2-\cancel{3x}+\cancel{2ix}-\cancel{3x}+9-\cancel{6i}-\cancel{2ix}+\cancel{6i}-\overset{+4}{4i^2})$

$a(x+2)(x-5)(x^2-6x+13)$

$a(3)(-4)(1-6+13) = a(-12)(8) = -96$

$-96a = -96$

$a = 1$

$f(x) = (x+2)(x-5)(x^2-6x+13)$

6. Sketch the graph of the function $f(x) = \frac{x^3-1}{x^2-9}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (15 points)

$\frac{(x-1)(x^2+x+1)}{(x-3)(x+3)}$

$x = 1$
x-int

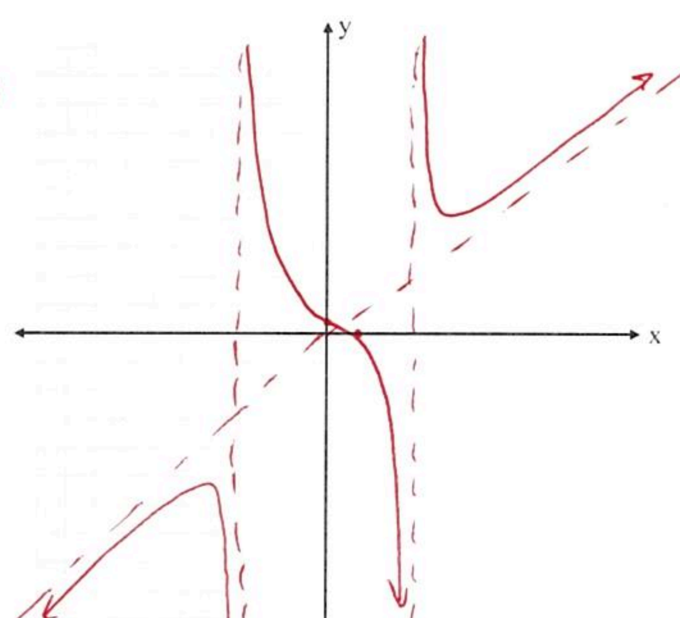
VA: $x=3, x=-3$

SA

$$\begin{array}{r|l} & x \\ x^2-9 & x^3+0x^2+0x-1 \\ & -x^3 \quad +9x \\ \hline & 9x-1 \end{array}$$

$y = x$

y-int
$x=0$ $\frac{-1}{-9} = \frac{1}{9}$



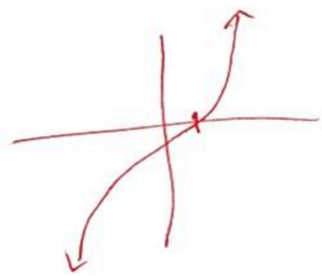
7. Solve the rational and polynomial inequalities and write the solution in interval notation. Be sure to create a sign chart. You should consider checking your answer with a graph. (15 points each)

a. $x^3 - x^2 + 9x - 9 > 0$

$x^2(x-1) + 9(x-1) > 0$

$(x^2+9)(x-1) > 0$

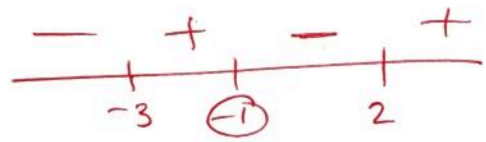
$(1, \infty)$



b. $\frac{(x+3)(x-2)}{x+1} \leq 0$

$x=0$

$\frac{(3)(-2)}{(1)} = -6$



$(-\infty, -3] \cup (-1, 2]$