2/17/2022

3.9 Exponential and Logarithmic Differentiation (Logarithmic differentiation)
1.5 Review of the Algebra of Hyperbolic Trig Functions
6.9 (part) Derivatives of Hyperbolic Trig Functions

--Single variable differentiation review: we can now do everything except questions 23, 24. Rest are good practice for the exam

Logarithmic differentiation.
Used to differentiate functions like:

$$y = [f(x)]^{g(x)}$$

Example.

$$f(x) = x^x$$

$$y = x^x$$

Apply the natural log to both sides of the equation.

$$\ln y = \ln(x^x)$$

Apply logarithm rules to remove the x in the exponent and multiply in front.

$$\ln y = x \ln x$$

Take the derivative of the resulting function implicitly.

$$\frac{1}{y}y' = 1(\ln x) + x \cdot \frac{1}{x}$$

$$\frac{1}{y}y' = \ln x + 1$$

Solve for y'

$$y' = y(\ln x + 1)$$

Replace y with the original function.

$$y' = x^x(\ln x + 1)$$

Example.

$$f(x) = (\sin e^x)^{x^2}$$

$$y = (\sin e^x)^{x^2}$$

$$\ln y = \ln\left[(\sin e^x)^{x^2}\right]$$

$$\ln y = x^2 \ln(\sin e^x)$$

$$\frac{1}{y}y' = 2x \ln(\sin e^x) + x^2 e^x \cot e^x$$

x^2	$\ln(\sin e^x)$

$2x$	$\frac{1}{\sin e^x}\cos(e^x)\, e^x = \frac{e^x \cos e^x}{\sin e^x} = e^x \cot e^x$

$$y' = y(2x \ln(\sin e^x) + x^2 e^x \cot e^x)$$

$$y' = (\sin e^x)^{x^2}[2x \ln(\sin e^x) + x^2 e^x \cot e^x]$$

Example.

$$f(x) = \ln(\tan x^3)^{\sqrt{x}} = \ln^{\sqrt{x}}(\tan x^3)$$

$$y = (\ln(\tan x^3))^{\sqrt{x}}$$

$$\ln y = \ln\left[(\ln(\tan x^3))^{\sqrt{x}}\right]$$

$$\ln y = \sqrt{x} \ln[\ln(\tan x^3)]$$

$$\frac{1}{y} y' = \frac{1}{2\sqrt{x}} \ln[\ln(\tan x^3)] + \sqrt{x} \frac{3x^2 \sec^2 x^3}{\tan x^3 \cdot \ln(\tan x^3)}$$

$\sqrt{x} = x^{1/2}$	$\ln[\ln(\tan x^3)]$
$\frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$	$\frac{1}{\ln(\tan x^3)} \cdot \frac{1}{\tan x^3} \cdot \sec^2 x^3 \cdot 3x^2$ $= \frac{3x^2 \sec^2 x^3}{\tan x^3 \cdot \ln(\tan x^3)}$

$$\frac{1}{y} y' = \frac{\ln[\ln(\tan x^3)]}{2\sqrt{x}} + \frac{3x^{5/2} \sec^2 x^3}{\tan x^3 \cdot \ln(\tan x^3)}$$

$$y' = y\left[\frac{\ln[\ln(\tan x^3)]}{2\sqrt{x}} + \frac{3x^{5/2} \sec^2 x^3}{\tan x^3 \cdot \ln(\tan x^3)}\right]$$

$$y' = (\ln(\tan x^3))^{\sqrt{x}}\left[\frac{\ln[\ln(\tan x^3)]}{2\sqrt{x}} + \frac{3x^{5/2} \sec^2 x^3}{\tan x^3 \cdot \ln(\tan x^3)}\right]$$

Hyperbolic Trig Functions

Regular vs. Hyperbolic Trigonometric Functions

Regular	Hyperbolic
$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\cos x = \frac{e^{ix} + e^{-ix}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tan x = \frac{\sin x}{\cos x} = \frac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$	$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$\cot x = \frac{i(e^{ix} + e^{-ix})}{e^{ix} - e^{-ix}}$	$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
$\sec x = \frac{2}{e^{ix} + e^{-ix}}$	$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$
$\csc x = \frac{2i}{e^{ix} - e^{-ix}}$	$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$

The regular trig functions follow from Euler's formula: $e^{ix} = \cos x + i \sin x$.

The identity rules are based on hyperbolas.

$$\cos^2 x + \sin^2 x = 1$$
$$\cosh^2 x - \sinh^2 x = 1$$

A note on pronunciation:
Sinh = sinch
Cosh = cosh
Tanh = tanch
Coth = coth or cotanch
Sech = seetch
Csch = co-seetch

Derivatives of hyperbolic trig functions.

$$f(x) = \sinh x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}(e^x - e^{-x})$$

$$f'(x) = \frac{1}{2}\left(e^x - (-e^{-x})\right) = \frac{1}{2}(e^x + e^{-x}) = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx}[\sinh x] = \cosh x$$

$$\frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$$

$$\frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Trigonometric

Derivatives:

$$\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2}$$

$$\frac{d}{dx}[\coth^{-1} x] = \frac{1}{1-x^2}$$

$$\frac{d}{dx}[\text{sech}^{-1} x] = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\text{csch}^{-1} x] = -\frac{1}{|x|\sqrt{1+x^2}}$$

End of the First exam.

That exam is March 1st.
Thursday 2/24 – we will do a review in class – bring lots of questions.