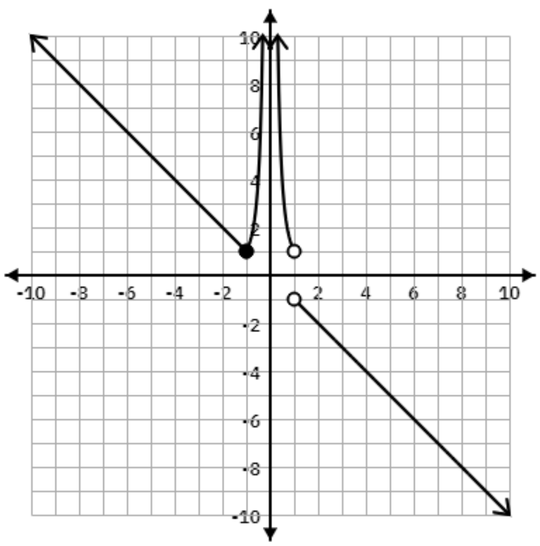
2/24/2022

Review for Exam #1
More examples from 4.1 Related Rates
4.2 Differentials and linear approximations

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{1}{|u|\sqrt{(u)^2 - 1}}u'$$

Requests: limits with piecewise functions
log/ln rules + exponential/log derivatives
Implicit differentiation



$$f(x) = \begin{cases} -x, & x \le -1 \\ \frac{1}{x^2}, & -1 < x < 1 \\ -x, & x > 1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = -1$$
$$\lim_{x \to 1^-} f(x) = 1$$
$$\lim_{x \to 1} f(x) = DNE$$

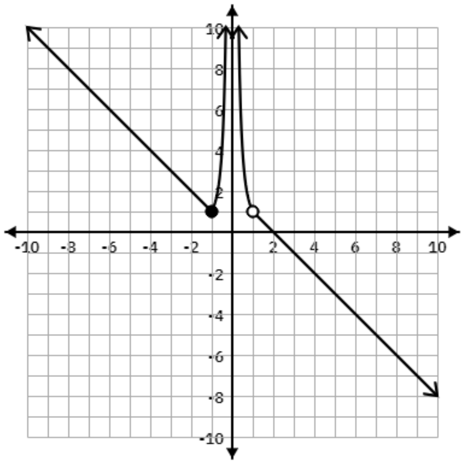
Is the function continuous at x=1? No. What kind of discontinuity is it? A jump discontinuity.
(for limits, use “does not exist” or DNE, you should not say “undefined”)

$$\lim_{x \to -1^+} f(x) = 1$$
$$\lim_{x \to -1^-} f(x) = 1$$
$$\lim_{x \to -1} f(x) = 1$$

Is the function continuous at x=-1? Yes. Explain. The limits are the same from both sides, the function has to be defined at the point, and the limit and the function have to agree.

$$\lim_{x \to 0^+} f(x) = \infty$$
$$\lim_{x \to 0^-} f(x) = \infty$$
$$\lim_{x \to 0} f(x) = \infty$$

Is the function continuous at x=0? No (infinity is not a number). Infinite discontinuity.



$$(x) = \begin{cases} -x, & x \leq -1 \\ \frac{1}{x^2}, -1 < x < 1 \\ -x + 2, & x > 1 \end{cases}$$

X=1 is a removable discontinuity.

Exponential/log rules

$$e^{(x+y)} = e^x e^y$$
$$e^{\ln x} = x \text{ in the context of } a^x = e^{x \ln a}$$
$$\log(MN) = \log M + \log N$$
$$\log \frac{M}{N} = \log M - \log N$$
$$\log M^r = r \log M$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[a^x] = (\ln a)a^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{\ln a} \cdot \frac{1}{x}$$

Find the derivative of $f(x) = e^{x^3 \ln x}$.

$$f(u) = e^u, u = x^3 \ln x$$

$$f'(x) = f'(u)u' = e^u u' = e^{x^3 \ln x} u'$$

x^3	$\ln x$

$3x^2$	$\frac{1}{x}$

$$f'(x) = e^{x^3 \ln x}\left(3x^2 \ln x + x^3 \cdot \frac{1}{x}\right) = e^{x^3 \ln x}(3x^2 \ln x + x^2)$$

$$g(t) = 3^{4t}$$
$$g(u) = 3^u, u = 4t$$

$$g'(t) = g'(u)u' = (\ln 3)3^u u' = (\ln 3)3^{4t}4 = (4 \ln 3)3^{4t}$$

$$h(x) = x^{\ln x}$$

$$y = x^{\ln x}$$
$$\ln y = \ln(x^{\ln x})$$
$$\ln y = (\ln x)(\ln x) = (\ln x)^2$$
$$\frac{1}{y}y' = 2(\ln x)\frac{1}{x}$$

$$y' = \frac{2y \ln x}{x}$$

$$y' = \frac{2x^{\ln x} \ln x}{x}$$

Implicit differentiation

$$x^3y + xy^3 = -8$$

$$3x^2y + x^3y' + y^3 + 3xy^2y' = 0$$
$$x^3y' + 3xy^2y' = -3x^2y - y^3$$
$$y'(x^3 + 3xy^2) = -3x^2y - y^3$$

$$y' = \frac{-3x^2y - y^3}{x^3 + 3xy^2}$$

$$x \tan y + e^{xy} - \ln(x + y) = 0$$

$$\tan y + x \sec^2 y\, y' + e^{xy}(y + xy') - \frac{1}{x + y}(1 + y') = 0$$

$$\tan y + x \sec^2 y\, y' + ye^{xy} + xe^{xy}y' - \frac{1}{x + y} - \frac{y'}{x + y} = 0$$

$$x \sec^2 y\, y' + xe^{xy}y' - \frac{y'}{x + y} = \frac{1}{x + y} - \tan y - ye^{xy}$$

$$y'\left(\sec^2 y + xe^{xy} - \frac{1}{x+y}\right) = \frac{1}{x+y} - \tan y - ye^{xy}$$

$$y' = \frac{\frac{1}{x+y} - \tan y - ye^{xy}}{\sec^2 y + xe^{xy} - \frac{1}{x+y}}$$

Linear approximations (3.3 or 3.4)

Suppose I want to estimate the value of 2.1^3. Use a linear approximation (then compare your answer to the true value).

$$f(x) = x^3$$

Use a nice value (like $x = 2$) and use it to estimate the messy value nearby. $\Delta x = h = 0.1$

$$f(2) = y$$

$f'(2)$ which is the slope of tangent line (the derivative of the function at the nice point).

$$f'(x) = 3x^2, f'(2) = 12$$

$$\Delta y = f'(x)\Delta x$$

$$y + \Delta \text{y} = f(x) + f'(x)\Delta x$$

$$\Delta y = 12(0.1) = 1.2$$

$$f(2.1) \approx 2^3 + 1.2 = 9.2$$

9.2 is the estimate (linear estimate) for 2.1^3. How good is it? True value is 9.261. Not bad.

The one on the exam, the answer is a lot closer to the true value and you'll need lots of decimal places.