MTH 263, Quiz #11, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

$$\sum^{n} i = \frac{n(n+1)}{2} \qquad \sum^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. Use summation formulas to find the value of the sums.

a. $\sum_{i=1}^{10}(i^2 - i) = \sum^{10} i^2 - \sum^{10} i$

$$\frac{10(10+1)(20+1)}{6} - \frac{10(10+1)}{2} = \boxed{330}$$

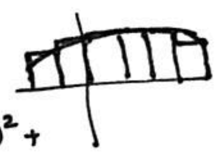
b. $\sum_{j=11}^{20}(j^3 - 2j) = \sum_{j=1}^{20}(j^3 - 2j) - \sum_{j=1}^{10}(j^3 - 2j) = \sum^{20} j^3 - 2\sum^{20} j - \sum^{10} j^3 + 2\sum^{10} j =$

$$\frac{20^2(20+1)^2}{4} - 2\left[\frac{20(20+1)}{2}\right] - \frac{10^2(10+1)^2}{4} + 2\left[\frac{10(10+1)}{2}\right] =$$

$$44{,}100 - 420 - 3025 + 110 = \boxed{40{,}765}$$

2. Estimate the area under the curve $f(x) = 10 + 3x - x^2$ on the interval [-1,2] using 6 rectangles and the right-endpoint rule. Sketch a graph of what you are doing.

$\Delta x = \frac{2-(-1)}{6} = \frac{3}{6} = \frac{1}{2}$



$x_0 = -1,\ x_1 = -\frac{1}{2},\ x_2 = 0,\ x_3 = \frac{1}{2},\ x_4 = 1,\ x_5 = \frac{3}{2},\ x_6 = 2$

$$\sum_{i=1}^{6} f(x_i)\Delta x = \frac{1}{2}\Big[10 + 3(-\tfrac{1}{2}) - (-\tfrac{1}{2})^2 + 10 + 3(0) - 0^2 + 10 + 3(\tfrac{1}{2}) - (\tfrac{1}{2})^2 + 10 + 3(1) - (1)^2 + 10 + 3(\tfrac{3}{2}) - (\tfrac{3}{2})^2 + 10 + 3(2) - 2^2\Big] =$$

$$\frac{1}{2}[65.75] = 32.875 \qquad x_i = -1 + \frac{i}{2}$$

$$\sum_{i=1}^{6} f(x_i)\Delta x = \frac{1}{2}\sum^{6}\left[10 + 3\left(-1 + \frac{i}{2}\right) - \left(-1 + \frac{i}{2}\right)^2\right] = \frac{1}{2}\sum^{6}\left[10 - 3 + \frac{3i}{2} - \left(1 - i + \frac{i^2}{4}\right)\right] =$$

$$\frac{1}{2}\sum^{6}\left[6 + \frac{5i}{2} - \frac{i^2}{4}\right] = \frac{1}{2}\left[6n + \frac{5n(n+1)}{4} - \frac{1}{4}\frac{n(n+1)(2n+1)}{6}\right]_{n=6} = \frac{1}{2}\left[36 + \frac{210}{4} - \frac{546}{24}\right] = 32.875$$

3. Find the exact value of the area under the curve on the same interval as in #2.

$$\int_{-1}^{2} 10 + 3x - x^2\,dx = 31.5$$

$\Delta x = \frac{3}{n} \quad x_i = -1 + \frac{3i}{n}$

$10 + 3\left(-1 + \frac{3i}{n}\right) - \left(-1 + \frac{3i}{n}\right)^2$

$10 - 3 + \frac{9i}{n} - \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)$

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum^{6}\left[6 + \frac{15i}{n} - \frac{9i^2}{n^3}\right]\frac{3}{n} = \lim_{n\to\infty}\frac{3}{n}\left[6n + \frac{15}{n}\frac{n(n+1)}{2} - \frac{9}{n^2}\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[18 + \frac{45(n^2+n)}{2n^2} - \frac{27}{6n^3}(n^2+n)(2n+1)\right] = \lim_{n\to\infty}\left[18 + \frac{45}{2} + \cancel{\frac{45}{2n}} - \frac{27\cdot 2n^3}{6n^3} - \cancel{\frac{27\cdot 3n^2}{6n^3}} - \cancel{\frac{27n}{6n^3}}\right] =$$

$(n^2+n)(2n+1) = 2n^3 + n^2 + 2n^2 + n$

$$18 + \frac{45}{2} - \frac{54}{6} = \boxed{31.5}$$

4. Explain why $\int_{-2}^{2} x^5 - x + \sin x\,dx$ is equal to zero without doing any integrating.

because the limits are $-a$ and a & the function is odd (symmetric to the origin) the positive area and negative area are equal and cancel out.