MTH 263, Quiz #3, Spring 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Using the formal $\varepsilon - \delta$ definition of the limit, prove that $\lim_{x\to4} 3x - 7 = 5$.

$|x-4| < \delta \quad |3x-7-5| < \varepsilon \to |3x-12| < \varepsilon \to 3|x-4| < \varepsilon \to 3\delta < \varepsilon \to \delta < \varepsilon/3$

Suppose $|x-4| < \delta$, then if we let $\delta = \varepsilon/3$, This implies that $|x-4| < \varepsilon/3$ and that $3|x-4| < \varepsilon$, and that $|3x-12| < \varepsilon$. We can rewrite this as $|(3x-7)-5| < \varepsilon$ which is what we wanted to show.

2. Use the formal definition of the derivative to find a formula for the derivative $f'(x)$ for the function $f(x) = 2x^2 + 4x - 11$. Then use that formula to find the slope of the tangent line at the point $x = 2$, and then write the equation of the tangent line at that point.

$$f'(x) = \lim_{h\to0} \frac{2(x+h)^2 + 4(x+h) - 11 - (2x^2+4x-11)}{h} =$$

$$\lim_{h\to0} \frac{2(x^2+2xh+h^2) + 4x + 4h - 11 - 2x^2 - 4x + 11}{h} =$$

$$\lim_{h\to0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{4x} + 4h - \cancel{11} - \cancel{2x^2} - \cancel{4x} + \cancel{11}}{h} =$$

$$\lim_{h\to0} \frac{4xh + 2h^2 + 4h}{h} = \lim_{h\to0} \frac{\cancel{h}(4x+2h+4)}{\cancel{h}} = \lim_{h\to0} 4x + 2h + 4 =$$

$$\boxed{4x+4}$$

$f(2) = 2(2)^2 + 4(2) - 11 =$
$8 + 8 - 11 = 16 - 11 = 5$

$f'(2) = 4(2) + 4 = 8 + 4 = 12$

$m = 12$, pt $(2,5)$

$$\boxed{y - 5 = 12(x-2)}$$

$y - 5 = 12x - 24$

$$\boxed{y = 12x - 19}$$