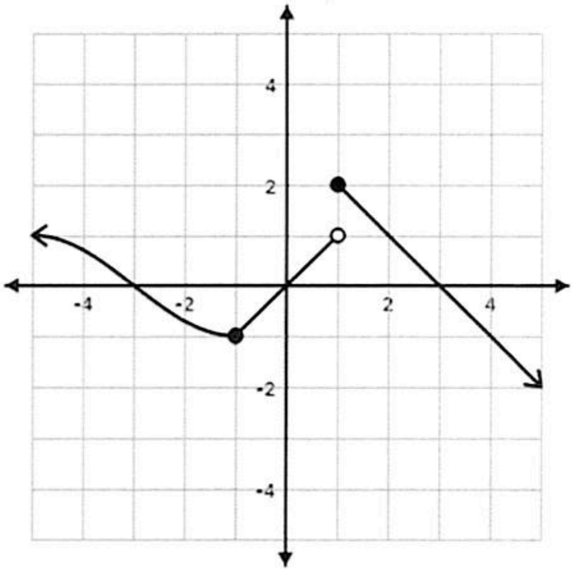
MTH 261, Exam #1, Spring 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. Use the graph below to answer the following questions. (3 points each)



a. $\lim\limits_{x\to -1^+} F(x)$ $= -1$

b. $\lim\limits_{x\to -1^-} F(x)$ -1

c. $\lim\limits_{x\to 1^+} F(x)$ $= 2$

d. $\lim\limits_{x\to 1^-} F(x)$ $= 1$

e. $\lim\limits_{x\to 1} F(x)$ DNE

f. $\lim\limits_{x\to -1} F(x)$ $= -1$

g. $F(-1)$ $= 1$

h. $F(1)$ $= 2$

2. Use the first derivative test (create a sign chart) to determine whether each of the critical points of the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 4x - 8$ is a maximum or a minimum (or neither). (12 points)

$f'(x) = x^2 - 4x + 4 = 0 \quad (x-2)^2 = 0 \to x = 2$

$f''(x) = 2x - 4 = 0$

$f''(2) = 0$ inflection pt.

+ +
2

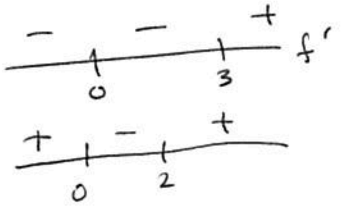
− +
2

neither

3. Use the second derivative test to determine whether each of the critical points of the graph $g(x) = x^4 - 4x^3 - 5$ is a maximum or a minimum (or neither). (10 points)

$g'(x) = 4x^3 - 12x^2 = 0 \quad 4x^2(x-3) = 0$ $\quad x = 0,\ x = 3$

$g''(x) = 12x^2 - 24x = 0 \quad 12x(x-2) = 0$ $\quad x = 0,\ x = 2$



x=0 is neither

x = 3 minimum

4. Evaluate $\lim_{x \to \cancel{25}} \frac{4-\sqrt{x}}{x-16}$ numerically. Show a table of values to support your evaluation of the limit. (8 points) 16

$$\lim_{x \to 16} \frac{-(\cancel{\sqrt{x}-4})}{(\cancel{\sqrt{x}-4})(\sqrt{x}+4)} = \frac{-1}{\sqrt{x}+4} = -\frac{1}{8}$$

16.00001	16.0001	16.001	16.01	16.1	17
−.125	−.125	−.125	−.125	−.1248	−.1231
15.99999	15.9999	15.999	15.99	15.9	15
−.125	−.125	−.125	−.125	−.1252	−.127

5. In t seconds, an object dropped from a certain height will fall $s(t)$ feet, where $s(t) = 16t^2$.

   a. Find $s(1), s(4)$. (4 points)

   $s(1) = 16$

   $s(4) = 4096$

   b. Find the average rate of change (or average speed) over this period from 1 seconds to 4 seconds. (6 points)

$$\frac{4096-16}{4-1} = \frac{4080}{3} \approx 1360$$

6. Find the critical point(s) and determine whether each is a maximum, minimum, saddle point, or cannot be determined, for the function $f(x,y) = x^2 + xy + y^2 - 5y$. (10 points)

$f_x = 2x + y = 0$ $y = -2x$ $\to y = +10/3$ $(-5/3,\ 10/3)$ minimum

$f_y = x + 2y - 5 = 0$ $x - 4x = 5$

$-3x = 5$

$x = -5/3$

$f_{xx} = 2$

$f_{xy} = 1$ $D = (2)(2) - 1^2 = 3 > 0$ $f_{xx} > 0\ \cup$

$f_{yy} = 2$

7. Find the value of $f(x,y) = x^2 - y^3 + xy$ for $f(-2,1)$, and $f(3,-4)$. (8 points)

$f(-2,1) = (-2)^2 - 1^3 + (-2)(1) = 4 - 1 - 2 = 1$

$f(3,-4) = (3)^2 + (-4)^3 + (3)(-4) = 9 + 64 - 12 = 58$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

8. Use the definition of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of the following functions. (8 points each)

a. $f(x) = -7x + 5$

$$\lim_{h \to 0} \frac{-7(x+h) + 5 - (-7x+5)}{h} = \lim_{h \to 0} \frac{\cancel{-7x} - 7h \cancel{+5} + \cancel{7x} \cancel{-5}}{h} = -7$$

b. $f(x) = 2x^2 + 3x + 9$

$$\lim_{h \to 0} \frac{2(x+h)^2 + 3(x+h) + 9 - (2x^2 + 3x + 9)}{h} = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + h^2 \cancel{+3x} + 3h \cancel{+9} \cancel{-2x^2} \cancel{-3x} \cancel{-9}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + h^2 + 3h}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + h + 3)}{\cancel{h}} = 4x + 3$$

9. Find the equation of the tangent line at $x = 1$ of the graph $f(x) = 2\sqrt[4]{x}$. (12 points)

$2x^{1/4}$

$f'(x) = 2 \cdot \frac{1}{4} x^{-3/4}$ $\quad f'(1) = \frac{1}{2} \cdot 1 = \frac{1}{2}$ $\quad f(1) = 2$

$y - 2 = \frac{1}{2}(x - 1)$

10. Find the derivative of $f(x) = \left(\ln(2x - 1) + e^{x^3+4}\right)^5$. (10 points)

$f'(x) = 5\left(\ln(2x-1) + e^{x^3+4}\right)^4 \cdot \left(\frac{2}{2x-1} + 3x^2 e^{x^3+4}\right)$

11. Find f_{xy} and f_{yy} for the function $f(x, y) = x^3y^3 - x^2y^2 + x + 2y$. (10 points)

$f_x = 3x^2y^3 - 2xy^2 + 1$

$f_{xy} = 9x^2y^2 - 4xy$

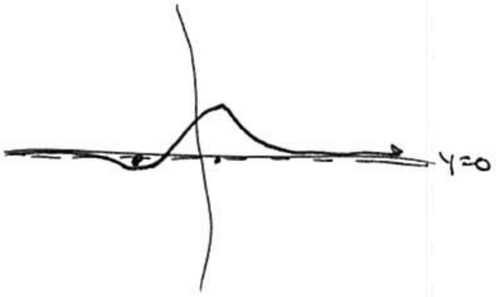
$f_y = 3x^3y^2 - 2x^2y + 2$

$f_{yy} = 6x^3y - 2x^2$

12. Sketch the graph of the function $f(x) = \frac{x+1}{x^2+1}$ using calculus and algebraic techniques. Note any intercepts, critical points, and asymptotes. (16 points)

no vertical asymptotes

horizontal asymptote at y = 0



$$f'(x) = \frac{1(x^2+1) - 2x(x+1)}{(x^2+1)^2} = \frac{x^2+1-2x^2-2x}{(x^2+1)^2} =$$

$$= \frac{-x^2-2x+1}{(x^2+1)^2}$$

$$x = \frac{2 \pm \sqrt{4-4(1)(-1)}}{-2} = \frac{2 \pm \sqrt{8}}{-2} = \frac{-1 \pm \sqrt{2}}{2}$$

13. Find the derivative of the functions below. (8 points each)

a. $f(x) = \frac{5}{6x^4} - \sqrt[9]{x^5} = \frac{5}{6}x^{-4} - x^{5/9}$

$$f'(x) = -\frac{10}{3}x^{-5} - \frac{5}{9}x^{-4/9} = -\frac{10}{3x^5} - \frac{5}{9\sqrt[9]{x^4}}$$

b. $g(x) = e^{-2x} - \frac{9}{8}e^{x^2} + 7^x$

$$g'(x) = -2e^{-2x} - \frac{9x}{4}e^{x^2} + (\ln 7)7^x$$

c. $h(x) = [\ln(e - x^2)]^3$

$$h'(x) = 3[\ln(e-x^2)]^2 \cdot \frac{1}{e-x^2} \cdot 2x$$

d. $F(x) = \frac{5x^3+\sqrt{x}}{x^4+11x}$

$$F'(x) = \frac{\left(15x^2+\frac{1}{2\sqrt{x}}\right)(x^4+11x) - (4x^3+11)(5x^3+\sqrt{x})}{(x^4+11x)^2}$$

e. $G(x) = x^3\sqrt{9x^2+4}$

$$G'(x) = 3x^2\sqrt{9x^2+4} + x^3 \cdot \frac{1}{2}(9x^2+4)^{-1/2} \cdot 18x$$

14. Find the first 4 derivatives of $g(x) = x^5 - 2x^4 - 3x^3 - 7x^2 + \frac{9}{x}$. (8 points)

$g'(x) = 5x^4 - 8x^3 - 9x^2 - 14x - \frac{9}{x^2}$

$g''(x) = 20x^3 - 24x^2 - 18x - 14 + \frac{18}{x^3}$

$g'''(x) = 60x^2 - 48x - 18 - \frac{54}{x^4}$

$g^{iv}(x) = 120x - 48 + \frac{216}{x^5}$

15. Find the first partial derivatives of each of the functions. (8 points each)

a. $f(x, y) = 4x^2y^2 + 5xy - y^2$

$f_x = 8xy^2 + 5y$

$f_y = 8x^2y + 5y - 2y$

b. $g(x, y, z) = e^{xz} - z\ln(y + 5) + x^{3.2}y^{-0.8}$

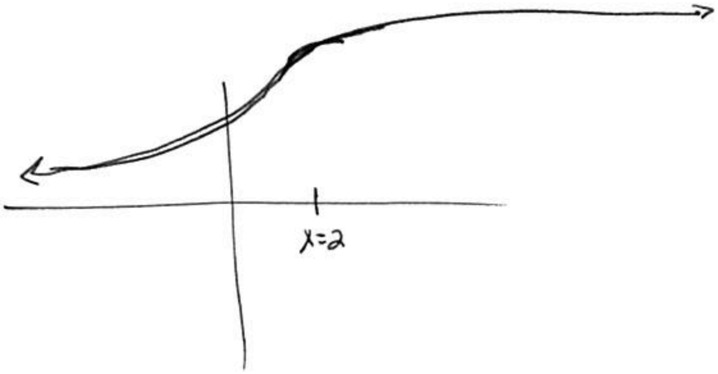
$$g_x = ze^{xz} + 3.2x^{2.2}y^{-0.8}$$

$$g_y = -\frac{z}{y+5} - 0.8x^{3.2}y^{-1.8}$$

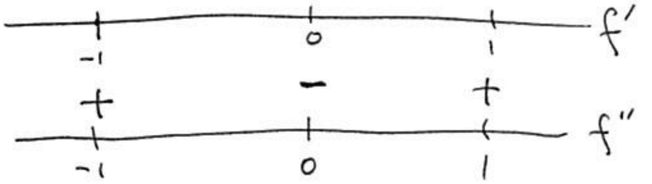
$$g_z = xe^{xz} - \ln(y+5)$$

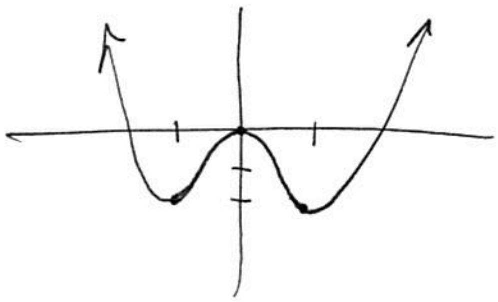
16. Sketch a graph of a function with the following properties: (6 points each)

a. f is increasing and concave up on $(-\infty, 2)$, and f is increasing and concave down on $(2, \infty)$.

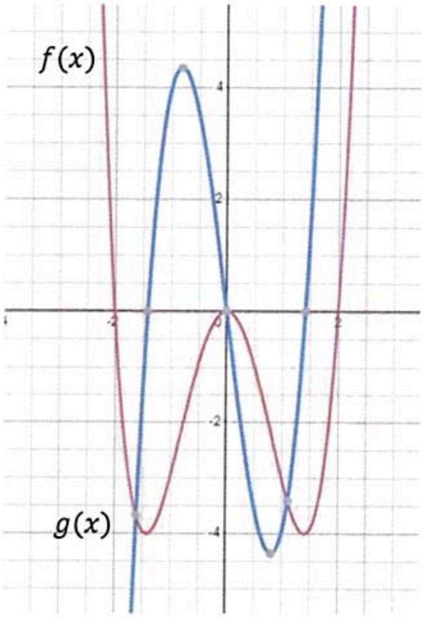


b. $f'(-1) = 0, f''(-1) > 0, f(-1) = -2; f'(1) = 0, f''(1) > 0, f(1) = -2;\ f'(0) = 0, f''(0) < 0, f(0) = 0.$





17. Identify, in the graph to the right, which function is the derivative, and which is the original function. (6 points)



red (f(x))
is the original

and blue g(x)
is the derivative