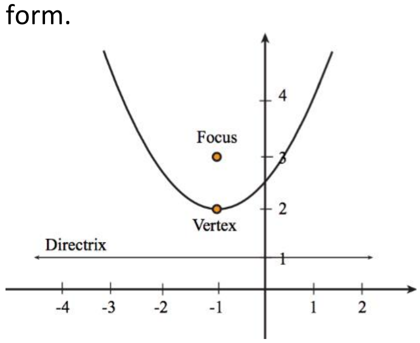
MTH 162, Homework #6, Spring 2025 Name ____

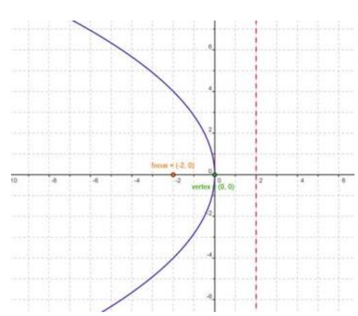
Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

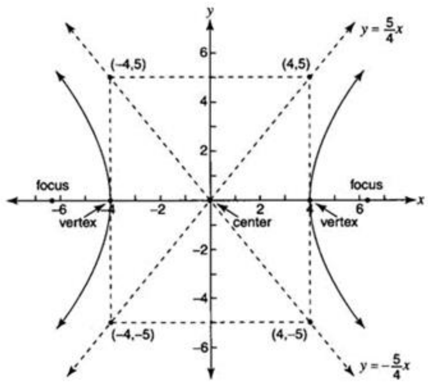
1. Graph the conic sections. Find and label any vertices, center, foci, asymptotes, directrix, etc.
   a. $\frac{x^2}{16}+\frac{y^2}{4}=1$
   b. $\frac{x^2}{8}-\frac{y^2}{25}=1$
   c. $y^2=-8x$
   d. $x^2=1-4y^2$
   e. $y=\pm\sqrt{x^2-3}$
   f. $\frac{(x-2)^2}{9}+\frac{(y-1)^2}{4}=1$
   g. $\frac{(x+3)^2}{25}-\frac{y^2}{16}=1$
   h. $(y+4)^2=12(x+2)$
   i. $(x-3)^2-4(y+3)^2=4$
   j. $4x^2+y^2+16x-6y-39=0$
   k. $9x^2-16y^2-36x-64y+116=0$
   l. $y^2-2y+12x-35=0$
   m. $\frac{x^2}{16}+\frac{y^2}{49}=1$
   n. $\frac{y^2}{25}-\frac{x^2}{64}=1$
   o. $4x^2+25y^2=100$
   p. $9y^2-25x^2=225$
   q. $8y^2+4x=0$
   r. $\frac{(x-1)^2}{2}+\frac{(y+3)^2}{5}=1$
   s. $(x-2)^2=8(y-1)$
   t. $(x-3)^2+4(y-2)^2=16$
   u. $9x^2+16y^2-18x+64y-71=0$
   v. $16x^2-y^2+64x-2y+67=0$
   w. $x^2+6x+8y+1=0$

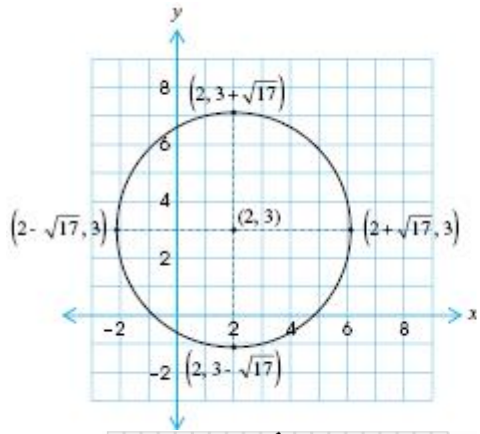
2. Graph the conic section with the given information and write the equation of the graph in standard form.
   a. Ellipse: foci $(-5,0),(5,0)$, vertices $(-8,0),(8,0)$
   b. Ellipse: foci $(0,-2),(0,2)$, x-intercepts $-2,2$
   c. Ellipse: major axis length 10, minor axis length 4, center $(-2,3)$
   d. Ellipse: vertices $(7,9),(7,3)$, minor axis endpoints $(5,6),(9,6)$
   e. Hyperbola: foci $(0,-3),(0,3)$, vertices $(0,-1),(0,1)$
   f. Hyperbola: endpoints of transverse axis $(0,-6),(0,6)$, asymptotes $y=\pm 2x$
   g. Hyperbola: center $(4,-2)$, focus $(7,-2)$, vertex $(6,-2)$
   h. Parabola: focus $(7,0)$, directrix $x=-7$
   i. Parabola: focus $(0,15)$, directrix $y=-15$
   j. Parabola: vertex $(5,-2)$, focus $(7,-2)$
   k. Parabola: focus $(2,4)$, directrix $x=-4$
   l. Parabola: focus $(7,7)$, directrix $y=-9$

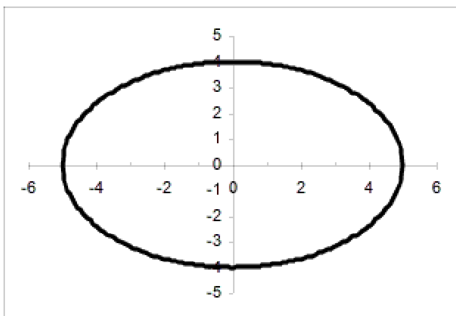
3. For each of the graphs of conic sections below, write the equation of the graph in standard form.

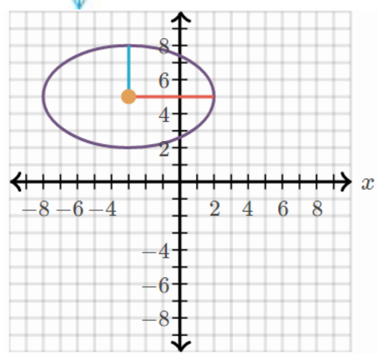


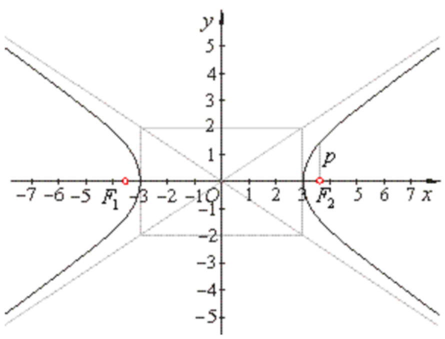












4. The elliptical ceiling in Statuary Hall in the U.S. Capitol Building is 96 ft long and 23 feet wide.
   a. If the origin is the center of the semi-elliptical room, write an equation for the shape of the dome.
   b. John Quincy Adams discovered that he could overhear the conversations of opposing party leaders near the left side of the chamber if he situated his desk at the focus on the right side of the chamber. How far from the center, along the major axis, did Adams situate his desk (to the nearest foot).

5. An Earth satellite has an elliptical orbit described by $\frac{x^2}{5000^2} + \frac{y^2}{4750^2} = 1$ with units in miles. The center of the Earth is at $(16,0)$.
   a. The perigee of the satellite's orbit is the point nearest to Earth's center. If the radius of the Earth is approximately 4000 miles, find the distance to the perigee above the Earth's surface.
   b. The apogee of the satellite's orbit is the points furthest from the Earth's center. Find the distance of the apogee above the surface.

6. Scattering experiments are experiments in which moving particles are deflected by various forces, led to the concept of the nucleus of an atom. In 1911, Ernest Rutherford discovered that when alpha particles are directed along hyperbolic paths. If a particle gets as close as 3 units to the nucleus along a hyperbolic path with asymptotes given by $y = \frac{1}{2}x$, what is the equation of its path?

7. The towers of a suspension bridge are 800 ft apart and rise 160 ft above the road. The cable between the towers has (approx.) the shape of a parabola, and the cable just touches the road midway between the towers. What is the height of the cable 100 ft from a tower?

8. Identify the conic section. Sketch the graph.

   a. $r = \frac{3}{1+\sin\theta}$ b. $r = \frac{12}{2-4\cos\theta}$ c. $r = \frac{15}{3-2\cos\theta}$ d. $r = \frac{12}{4+5\sin\theta}$