

Name KEY
 Math 255, Quiz #12, Summer 2012

Instructions: Show all work. Use exact answers.

1. Find the Laplace transform of the following functions:

a. $\mathcal{L}\{\sin(4t)\}$

$$\boxed{\frac{4}{s^2+16}}$$

b. $\mathcal{L}\{e^{-2t}\sin(4t)\}$

$$\boxed{\frac{4}{(s+2)^2+16}}$$

c. $\mathcal{L}\{t\sin(4t)\}$

$$(-1)^1 \frac{d}{ds} \left[\frac{4}{(s^2+16)^{-1}} \right] = -4 \left[-1 (s^2+16)^{-2} \cdot 2s \right] =$$

d. $\mathcal{L}\left\{f(t) = \begin{cases} \sin(4t), & 0 \leq t < \frac{\pi}{2} \\ 1, & t \geq \frac{\pi}{2} \end{cases}\right.$

$$f(t) = \sin 4t - \frac{\sin 4t}{s} + \mathcal{U}(t - \frac{\pi}{2}) + \mathcal{U}(t - \frac{\pi}{2})$$

$$\frac{4}{s^2+16} - e^{-\frac{\pi}{2}s} \frac{s}{s^2+16} + \frac{e^{-\frac{\pi}{2}s}}{s} \sin 4t = \cos \left[\frac{\pi}{2} - t \right] = \cos [4(t - \frac{\pi}{2})] = \cos 4(t - \frac{\pi}{2})$$

$$\boxed{\frac{4}{s^2+16} - e^{-\frac{\pi}{2}s} \frac{s}{s^2+16} + \frac{e^{-\frac{\pi}{2}s}}{s}}$$

e. $\mathcal{L}\{\sin(4t) * t^3\}$

$$\frac{4}{s^2+16} \cdot \frac{6}{s^4} = \frac{24}{s^4(s^2+16)}$$

2. Find the inverse Laplace transforms of the following functions:

a. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$

$$\boxed{\cos 3t}$$

b. $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2+9}\right\} = \frac{1}{3} \mathcal{L}^{-1}\left\{e^{-s} \cdot \frac{3}{s^2+9}\right\} = \frac{1}{3} \cos(3(t-1)) \mathcal{U}(t-1)$

c. $\mathcal{L}^{-1}\left\{\frac{s-7}{(s-3)^2+9}\right\}$

$$\frac{s-3-4}{(s-3)^2+9} = \frac{s-3}{(s-3)^2+9} - \frac{4}{(s-3)^2+9} = e^{3t} \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$$

$$- e^{3t} \frac{4}{3} \mathcal{L}^{-1}\left\{\frac{1 \cdot 3}{s^2+9}\right\} = \boxed{e^{3t} \cos 3t - \frac{4}{3} e^{3t} \sin 3t}$$