Name KEY

Math 255, Quiz #5, Summer 2012

Instructions: Show all work. Use exact answers except in word problems or when specifically asked to round.

1. Solve the Bernoulli equation $\frac{dy}{dx} + xy = xe^{x^2}y^{-4}$.

$n = -4$

$(1-n)y^{-n} = 5y^4$

$5y^4\frac{dy}{dx} + 5xy^5 = 5xe^{x^2}$

$z = y^{1-n} = y^5$

$z' = 5y^4\frac{dy}{dx}$

$z' + 5xz = 5xe^{x^2}$

$\mu = e^{\int 5x\,dx}$

$e^{5/2x^2}z' + 5xe^{5/2x^2}z = 5xe^{7/2x^2}$

$e^{5/2x^2}$

$\left(ze^{7/2x^2}\right)' = 5\int xe^{7/2x^2}dx$

$u = \frac{7}{2}x^2$

$du = 7x\,dx$

$ze^{5/2x^2} = \frac{5}{7}e^{\frac{7}{2}x^2} + C$

$z = \boxed{\frac{5}{7}e^{x^2} + Ce^{-7/2x^2} = y^5}$

2. I'd like to solve the differential equation $\frac{dy}{dx} = 2x^2 + y$, given the initial condition y(1)=1 for the value of the solution at y(2.5). I'd like to use 100 steps to get a good answer. What is the step size I should use? Then calculate just the first three steps of the procedure using that information.

$\frac{2.5-1}{100} = \frac{1.5}{100} = .015 = h$

$y_{n+1} = y_n + h\,f(x_n, y_n)$ $\quad x_0 = 1,\ y_0 = 1$

$y_1 = 1 + .015[2(1)^2 + 1] = 1.045$ $\quad x_1 = 1.015$

$y_2 = 1.045 + .015[2(1.015)^2 + 1.045] = 1.09158$ $\quad x_2 = 1.030$

$y_3 = 1.09158 + .015[2(1.030)^2 + 1.09158] = 1.13978\ldots$

$x_3 = 1.045$