Name KEY

Math 255, Quiz #9, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

1. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/sec, and if there is no damping, determine the position y of the mass at any time t. When does the mass first return to equilibrium? (i.e. when is y=0?)

$m = .1\text{kg}$ $\quad y(0)=0$ $\quad y'(0) = -.1$

$.1(9.8) = k(.05)$

$\frac{.98}{.05} = k$

$196 = k$

$.1y'' + 19.6y = 0$

$y'' + 196y = 0$

$r^2 + 196 = 0$

$r = \pm 14i$

$y = A\cos 14t + B\sin 14t$

$0 = A(1) \Rightarrow A = 0$

$y' = 14B\cos 14t$

$\frac{-.1}{14} = B \quad B = -\frac{1}{140}$

$y = -\frac{1}{140}\cos 14t$

$0 = -\frac{1}{140}\cos 14t$

$0 = \cos 14t$

$\frac{\pi}{2} = 14t \Rightarrow \boxed{\frac{\pi}{28} = t}$

2. Solve the Cauchy-Euler equation $t^2y'' + 5ty' + 4y = 0$.

$t^n = y$

$y' = nt^{n-1}$

$y'' = n(n-1)t^{n-2}$

$t^2(n)(n-1)t^{n-2} + 5tnt^{n-1} + 4t^n = 0$

$n(n-1)t^n + 5nt^n + 4t^n = 0$

$t^n[n^2 - n + 5n + 4] = 0$

$n^2 + 4n + 4 = 0$

$n = -2$ repeated

$y = t^{-2}, \ln t \cdot t^{-2}$

$\boxed{y = At^{-2} + Bt^{-2}\ln t}$