Math 2568, Quiz #13, Spring 2013 Name KEY

Instructions: Show all work. You must use exact answers for all solutions.

1. Diagonalize the matrix $A = \begin{bmatrix} 2 & 9 \\ 1 & 10 \end{bmatrix}$ through an appropriate similarity transformation. Be sure to clearly state the similarity transformation matrix used. Use this information to calculate A^4. Show all work.

$\begin{bmatrix} 2-\lambda & 9 \\ 1 & 10-\lambda \end{bmatrix} = (2-\lambda)(10-\lambda) - 9 = 20 - 12\lambda + \lambda^2 - 9 = \lambda^2 - 12\lambda + 11 = 0 \quad (\lambda - 11)(\lambda - 1) = 0$

$\lambda = 1, \lambda = 11$

$\lambda_1 = 1 \quad \begin{bmatrix} 1 & 9 \\ 1 & 9 \end{bmatrix} \quad \begin{matrix} x_1 = -9x_2 \\ x_2 = x_2 \end{matrix} \quad \begin{bmatrix} -9 \\ 1 \end{bmatrix} = \vec{v}_1 \qquad \lambda_2 = 11 \quad \begin{bmatrix} -9 & 9 \\ 1 & -1 \end{bmatrix} \quad \begin{matrix} x_1 = x_2 \\ x_2 = x_2 \end{matrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} -9 & 1 \\ 1 & 1 \end{bmatrix}$ ← transformation $\quad A = P^{-1} D P$

$D = \begin{bmatrix} 1 & 0 \\ 0 & 11 \end{bmatrix}$ ← diagonalized matrix

$A^4 = P^{-1} D^4 P = $

$= P^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 11^4 \end{bmatrix} P = \begin{bmatrix} 1465 & 1464 \\ 13176 & 13177 \end{bmatrix}$

2. Find the eigenvalues and eigenvectors of the matrix $B = \begin{bmatrix} -2 & 2 \\ -5 & 0 \end{bmatrix}$. Find a similarity transformation for this matrix that will transform the matrix into a rotation matrix. Factor out the appropriate scaling factor and give the angle of rotation.

$\begin{bmatrix} -2-\lambda & 2 \\ -5 & 0-\lambda \end{bmatrix} = (-2-\lambda)(-\lambda) + 10 = \lambda^2 + 2\lambda + 10 = 0 \qquad \lambda = \dfrac{-2 \pm \sqrt{4 - 4(10)}}{2} \quad -36 \to 6i$

$\lambda = -1 \pm 3i$

$\begin{bmatrix} -2-(-1+3i) & 2 \\ -5 & -(-1+3i) \end{bmatrix} = \begin{bmatrix} -1-3i & 2 \\ -5 & 1-3i \end{bmatrix} \qquad \begin{matrix} \cancel{-5}x_1 = \dfrac{(-1+3i)x_2}{-5} \\ x_2 = x_2 \end{matrix} \qquad \vec{v}_1 = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix} i$

$x_1 = \left(\frac{1}{5} - \frac{3}{5}i\right) x_2 \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} i$

$\lambda = -1 - 3i$

$P = \begin{bmatrix} 1 & 3 \\ 5 & 0 \end{bmatrix}$ transformation $\qquad C = \begin{bmatrix} -1 & -3 \\ 3 & -1 \end{bmatrix}$ rotation matrix $\qquad B = PCP^{-1}$

$\sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$

$C = \underset{\text{scaling factor}}{\sqrt{10}} \begin{bmatrix} -1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & -1/\sqrt{10} \end{bmatrix}$

$\cos\theta = -1/\sqrt{10} \quad \sin\theta = 3/\sqrt{10}$ 2nd Q angle

$\tan\theta = \frac{3}{-1} \quad \theta = \tan^{-1}(-3) + \pi$

$\theta \approx 1.89$ radians

or $\theta \approx 108°$