Math 2568, Quiz #15, Summer 2013 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. Consider the orthogonal basis for $\mathbb{R}^4$ given by $\left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ 2 \\ 1 \end{bmatrix} \right\}$. Find the representation of $\vec{v} = \begin{bmatrix} 1 \\ 6 \\ 3 \\ 2 \end{bmatrix}$ in that basis using the fact that the basis is an orthogonal basis.

$$c_1 = \frac{3-12+3-2}{9+4+1+1} = \frac{-8}{15}$$

$$c_2 = \frac{0+6+6+0}{0+1+4+0} = \frac{12}{5}$$

$$c_3 = \frac{1+0+0+6}{1+0+0+9} = \frac{7}{9}$$

$$c_4 = \frac{-3-24+6+2}{9+16+4+1} = \frac{-19}{30}$$

$$[\vec{V}]_B = \begin{bmatrix} -8/15 \\ 12/5 \\ 7/9 \\ -19/30 \end{bmatrix}$$

2. Use the Gram-Schmidt process on the basis for $\mathbb{R}^3$ given by $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix} \right\}$ to find an orthogonal basis for the space. Then turn that basis into an orthonormal one. [Hint: the orthonormal basis is likely to look very ugly with nasty square roots and such.]

$$\vec{b}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \vec{b}_2 = \vec{v}_2 - \mathrm{proj}_{\vec{v}_1}\vec{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} - \frac{2-3+5}{1+1+1}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} - \frac{4}{3}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 13/3 \\ 11/3 \end{bmatrix}$$

$$\vec{b}_2 = \begin{bmatrix} 2 \\ 13 \\ 11 \end{bmatrix} \quad \vec{b}_3 = \vec{v}_3 - \mathrm{proj}_{\vec{v}_1}\vec{v}_3 - \mathrm{proj}_{\vec{b}_2}\vec{v}_3 = \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix} - \frac{1-7+10}{1+1+1}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \frac{2+91+110}{4+169+121}\begin{bmatrix} 2 \\ 13 \\ 11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix} - \frac{4}{3}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \frac{203}{294}\ \frac{29}{42}\begin{bmatrix} 2 \\ 13 \\ 11 \end{bmatrix} = \begin{bmatrix} -12/7 \\ -9/14 \\ 15/14 \end{bmatrix} \quad \vec{b}_3 = \begin{bmatrix} -24 \\ -9 \\ 15 \end{bmatrix} \text{ or } \begin{bmatrix} -8 \\ -3 \\ 5 \end{bmatrix}$$

$$\text{Orthogonal} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 13 \\ 11 \end{bmatrix}, \begin{bmatrix} -24 \\ -9 \\ 15 \end{bmatrix} \right\}$$

$$\text{Orthonormal} = \left\{ \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{294} \\ 13/\sqrt{294} \\ 11/\sqrt{294} \end{bmatrix}, \begin{bmatrix} -8/\sqrt{98} \\ -3/\sqrt{98} \\ 5/\sqrt{98} \end{bmatrix} \right\}$$