

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question and give answers in the specified format.

1. The following problems refer to the system $\begin{cases} -3x_1 + 5x_2 - 4x_3 + 9x_4 + 7x_5 = 11 \\ 5x_1 + 8x_2 + x_3 - 2x_4 - 4x_5 = -11 \end{cases}$

a. Write the system as a matrix equation.

$$\begin{bmatrix} -3 & 5 & -4 & 9 & 7 \\ 5 & 8 & 1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 11 \\ -11 \end{bmatrix}$$

b. Write the system as an augmented matrix.

$$\left[\begin{array}{ccccc|c} -3 & 5 & -4 & 9 & 7 & 11 \\ 5 & 8 & 1 & -2 & -4 & -11 \end{array} \right]$$

c. Solve the system and write the solution in parametric form.

Steps may vary

$$rref = \left[\begin{array}{ccccc|c} 1 & 0 & 37/49 & -82/49 & -76/49 & -143/49 \\ 0 & 1 & -17/49 & 39/49 & 23/49 & 22/49 \end{array} \right]$$

$$\begin{aligned} x_1 &= -37/49 x_3 + 82/49 x_4 + 76/49 x_5 - 143/49 \\ x_2 &= +17/49 x_3 - 39/49 x_4 - 23/49 x_5 + 22/49 \\ x_3 &= x_3 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

* Note, you can scale the parameters, not the constants.

$$\Rightarrow \vec{x} = x_3 \begin{bmatrix} -37 \\ 17 \\ 49 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 82 \\ -39 \\ 0 \\ 49 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 76 \\ -23 \\ 0 \\ 0 \\ 49 \end{bmatrix} + \begin{bmatrix} -143/49 \\ 22/49 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2. Determine if $\vec{b} = \begin{bmatrix} -1 \\ 4 \\ 5 \end{bmatrix}$ is a linear combination of $\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

$$\left[\begin{array}{cc|c} 2 & 0 & -1 \\ 3 & 2 & 4 \\ -2 & 1 & 5 \end{array} \right] \Rightarrow rref \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

no, \vec{b} is not a linear combination of \vec{v}_1 & \vec{v}_2