Math 2568, Quiz #6, Summer 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Use the row-reducing method to find the determinant of the following matrix.

$$\begin{vmatrix} 4 & 7 & 9 & 8 \\ 0 & 7 & 9 & -9 \\ 5 & 4 & 6 & -6 \\ 2 & 0 & 0 & 3 \end{vmatrix}$$

$R_4 \to R_1$ (−1) $\begin{vmatrix} 2 & 0 & 0 & 3 \\ 0 & 7 & 9 & -9 \\ 5 & 4 & 6 & -6 \\ 4 & 7 & 9 & 8 \end{vmatrix}$ $\frac{1}{2} R_1 \to R_1$ (2) $\begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 7 & 9 & -9 \\ 5 & 4 & 6 & -6 \\ 4 & 7 & 9 & 8 \end{vmatrix}$ $-5 \;\; 0 \;\; 0 \;\; -15/2$

$-4 \;\; 0 \;\; 0 \;\; -6$

$-5R_1 + R_3 \to R_3$
$-4R_1 + R_4 \to R_4$
(1) $\begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 7 & 9 & -9 \\ 0 & 4 & 6 & -27/2 \\ 0 & 7 & 9 & 2 \end{vmatrix}$ $\frac{1}{7} R_2 \to R_2$
$-R_2 + R_4 \to R_4$
(7) $\begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 9/7 & -9/7 \\ 0 & 4 & 6 & -27/2 \\ 0 & 0 & 0 & 11 \end{vmatrix}$ $0 \;\; -4 \;\; -36/7 \;\; 36/7$

$0 \;\; -7 \;\; -9 \;\; 9$

$-4R_2 + R_3 \to R_3$
(1) $\begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 9/7 & -9/7 \\ 0 & 0 & 6/7 & -117/14 \\ 0 & 0 & 0 & 11 \end{vmatrix}$

$\det = (1)(1)(6/7)(11) = \frac{66}{7}$

Original $\det = \left(\frac{66}{\cancel{7}}\right)(1)(\cancel{7})(1)(2)(-1) = -132$

2. Determine whether the set $H = \left\{ \begin{bmatrix} a+b \\ c \\ b+d \end{bmatrix}, ab \geq 0, a, b, c, d \text{ real} \right\}$ is a vector space.

$H = \left\{ a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ Not a vector space.

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is in the space.

$k\begin{bmatrix} a+b \\ c \\ b+d \end{bmatrix} = \begin{bmatrix} ka+kb \\ kc \\ kb+kd \end{bmatrix}$ ka, kb, kc, kd real,
$ka * kb = k^2 ab \geq 0$ yes if $ab \geq 0$

$\begin{bmatrix} a+b \\ c \\ b+d \end{bmatrix} + \begin{bmatrix} e+f \\ g \\ f+h \end{bmatrix} = \begin{bmatrix} (a+e)+(b+f) \\ c+g \\ (b+f)+(d+h) \end{bmatrix}$ is it true that if $ab \geq 0$ and $ef \geq 0$ that $(a+e)(b+f) \geq 0$? No

$a=8$ $b=1$, but $e=-1$, $f=-8$ $\Rightarrow (7)(-7) \geq 0$