Math 2366, Quiz #4, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Determine if the function $f(x) = \frac{x+1}{x^2+1}$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$. Justify your answer.

it is not. $f(0)=1$ and $f(1)=1$

not one-to-one.

also not onto since $f(x) \leq \sqrt{2}$

2. Give an explicit formula for a function from $\mathbb{Z}$ to $\mathbb{Z}^+$ that is:
   a. One-to-one but not onto.

$$f(n) = \begin{cases} 4n+1 & n \geq 0 \\ |4n+3| & n < 0 \end{cases}$$

   b. Onto, but not one-to-one.

$$f(n) = |n|$$

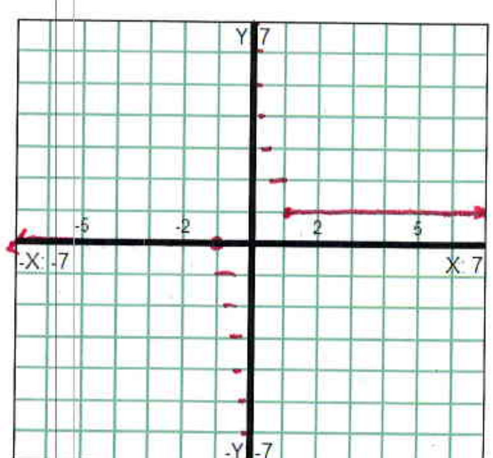
   c. One-to-one and onto

$$f(n) = \begin{cases} 2n & n \geq 0 \\ |2n+1| & n < 0 \end{cases}$$

   d. Neither one-to-one, nor onto.

$$f(n) = n^2$$

3. Draw the graph of the function $f(x) = \lceil 1/x \rceil$.



4. Find the first five terms of each sequence.

a. $\{a_n\}_{n=0} = n! - 2^n$

$\{0, -1, -2, -2, 8, 88 \ldots\}$

$a_0 = 1 - 1 = 0$ $a_2 = 2 - 4 = -2$ $a_4 = 24 - 16 = 8$

$a_1 = 1 - 2 = -1$ $a_3 = 6 - 8 = -2$ $a_5 = 120 - 32 = 88$

b. $a_n = -2a_{n-1}^2 + (n-1), a_0 = 2$

$\{2, -8, -127, -32256, -2080899069 \ldots\}$

$a_1 = -2(2)^2 + 0 = -8$

$a_2 = -2(-8)^2 + 1 = -128 + 1 = -127$

$a_3 = -2(-127)^2 + 2 = -32256$

$a_4 = -2(-32256)^2 + 3 = -2080899069$

c. $S_n = \sum_{i=1}^{n}(1 + (-1)^i)$

$S_1 = 1 - 1 = 0$ $S_4 = 1 + 1 = 2 + 2 = 4$

$S_2 = 1 + 1 = 2 + 0 = 2$ $S_5 = 1 - 1 = 0 + 4 = 4$

$S_3 = 1 - 1 = 0 + 2 = 2$

$\{0, 2, 2, 4, 4, \ldots\}$

5. Find a formula for the nth term of the sequence.

a. $3, 6, 12, 24, 48, 96, 192, \ldots$

$a_n = 3(2)^n$ n=0 start $\{a_n\}_{n=0} = 3 \cdot 2^n$

b. $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \ldots$

$a_n = 7 + 4n$ (start at $n=0$) $\{a_n\}_{n=0} = 7 + 4n$

c. $\frac{3}{1}, \frac{8}{5}, \frac{15}{23}, \frac{24}{119}, \frac{25}{719}, \frac{48}{5039}, \frac{63}{40319}, \frac{80}{362879}, \ldots$

$\frac{n^2 - 1}{(n-1)!} = \{a_n\}_{n=2}$