

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Give an example of two uncountable sets A and B such that $A - B$ is:

a. Finite

$$A = \mathbb{R} \quad B = \mathbb{R} - \{0\}$$

b. Countably infinite

$$A = \mathbb{R} \quad B = \mathbb{R} - \mathbb{Z}$$

answers will vary

c. Uncountable

$$A = \mathbb{R} \quad B = [0, 1]$$

2. Use mathematical induction to prove that $\sum_{k=0}^n k = \frac{n(n+1)}{2}$.

base case $n=1$ $\sum_{k=0}^1 k = 0+1=1$

induction: suppose that the formula is true for n .

$$\sum_{k=0}^{n+1} k = \sum_{k=0}^n k + (n+1) = \frac{n(n+1)}{2} + n+1 = \frac{n^2+n+2n+2}{2}$$

$$\frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} = \frac{(n+1)[(n+1)+1]}{2}$$

which is what is predicted by the formula.

QED.