

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Find the values of the following expressions by hand.

a. $P(8,5)$

$$\frac{8!}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$$

b. $C(12,6)$

$$\frac{12!}{6!6!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 11 \cdot 4 \cdot 3 = 132 \cdot 924$$

c. $\binom{5}{3}$

$$\frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 10$$

2. How many subsets with an odd number of elements does a set with 20 elements have?

$$\binom{20}{1} + \binom{20}{3} + \binom{20}{5} + \binom{20}{7} + \binom{20}{9} + \binom{20}{11} + \binom{20}{13}$$

$$+ \binom{20}{15} + \binom{20}{17} + \binom{20}{19} = 20 + 1140 + 15504 + 77520 +$$

$$167960 + 167960 + 77520 + 15504 + 1140 + 20 =$$

$$524,288$$

3. How many strings of the letters ABCDEFGH have the string BCA if repetition is not allowed.

$$P(6,6) = 6! = 720$$

4. Find the expansion of $(x + 2y)^5$

$$x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$$

5. If 1 10 45 120 210 252 210 120 45 10 1 is the 10th row in Pascal's triangle, use that information to find the 11th row.

1 11 55 165 330 462 462 330 165 55 11 1