Math 2415, Quiz #13, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Determine if the functions $f(x) = \sin(nx)$, $g(x) = \cos(nx)$ are orthogonal under the inner product $\int_{-\pi}^{\pi} f(x)g(x)dx$.

$u = \sin nx$

$\frac{1}{n} du = \cos nx$

$\frac{1}{n}\int u\, du = \frac{1}{2n}\sin^2 nx \Big|_{-\pi}^{\pi} = 0$

therefore, they are orthogonal

2. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}$.

$(3-\lambda)(4-\lambda) - 6 = \lambda^2 - 7\lambda + 6 = 0$

$(\lambda - 6)(\lambda - 1) = 0$

$\lambda = 6, \lambda = 1$

$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$ $3x_1 = 2x_2$ $\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \vec{v}_1$

$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$ $x_1 = -x_2$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \vec{v}_2$

3. Transform the second order differential equation $u^{IV} - u = 0$ into a system of linear differential equations.

$u = x_1$ $u' = x_2 = x_1'$ $u'' = x_3 = x_2'$ $u''' = x_4 = x_3'$ $u^{IV} = x_4'$

$u^{IV} = u$

$x_4' = x_1$

$x_3' = x_4$

$x_2' = x_3$

$x_1' = x_2$

$\Rightarrow$

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_4$

$x_4' = x_1$

$\Rightarrow \vec{x}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}}$