Math 2415, Quiz #3, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Solve the homogeneous first order equation $\frac{dy}{dx} = \frac{3y^2-x^2}{2xy}$.

$y = vx \qquad v = y/x$

$y' = v'x + v$

$v'x + v = \frac{3v^2x - x^2}{2x^2v} = \frac{3v^2-1}{2v}$

$-v \qquad\qquad -v$

$v'x = \frac{3v^2 - 1 - 2v^2}{2v} = \frac{v^2-1}{2v}$

$\int \left(\frac{2v}{v^2-1}\right) dv = \int \frac{1}{x}\, dx$

$\ln|v^2-1| = \ln x + C$

$= \ln Ax$

$v^2 - 1 = Ax$

$v^2 = Ax + 1$

$\frac{y^2}{x^2} = Ax + 1$

$y^2 = Ax^3 + x^2$

$y = \pm\sqrt{Ax^3 + x^2} = \pm x\sqrt{Ax+1}$

2. A tank contains 100 gallons of water and 50 oz of salt to start with. Water containing a brine solution with 85 oz of salt/gallon, and which flows into the tank at a rate of 2 gallons/minute. The well-mixed solution in the tank flows out at the same rate. Find an equation for the amount of salt in the tank at any time t.

$A(0) = 50$

$\frac{dA}{dt} = \text{Rate in} - \text{Rate out}$

$\frac{dA}{dt} = \frac{85\text{ oz}}{\text{gal}} \cdot \frac{2\text{ gal}}{\text{min}} - \frac{A\text{ oz}}{100\text{ gal}} \cdot \frac{2\text{ gal}}{\text{min}} \Rightarrow \frac{dA}{dt} = 170 - \frac{A}{50}$

$A' + \frac{A}{50} = 170 \qquad \frac{dA}{dt} = \frac{-1}{50}(8500 + A)$

$\int \frac{dA}{-8500+A} = \int \frac{-1}{50}\, dt \Rightarrow \ln|8500 + A| = -\frac{1}{50}t + C$

$A - 8500 = A_0 e^{-1/50 t} \Rightarrow A = 8500 + A_0 e^{-1/50 t}$

$A_0 = -8450$

$A(t) = 8500 - 8450\, e^{-1/50 t}$

3. Perform the indicated operations on the matrices/vectors given that $A = \begin{bmatrix} 2 & -1 \\ 5 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -3 & 7 \end{bmatrix}, \vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \vec{y} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$.

a. AB

$$\begin{bmatrix} 11 & -11 \\ 8 & 18 \end{bmatrix}$$

b. $B\vec{y}$

$$\begin{bmatrix} -44 \\ 55 \end{bmatrix}$$

c. $\vec{x}^T\vec{y}$

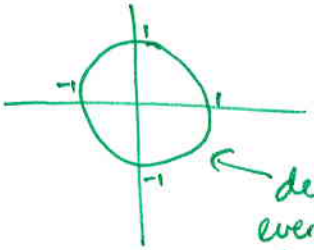
$-27+4 = -23$

4. Consider the nonlinear differential equation $y' = \frac{\cosh t}{1-t^2-y^2}$. Find any points in the plane at which the differential equation is not defined. Plot the region.

$\cosh t$ defined everywhere $(-\infty, \infty)$

$1 - t^2 - y^2 = 0 \Rightarrow 1 = t^2 + y^2$ undefined on unit circle

$\frac{\partial F}{\partial y} = \frac{\cosh t\,(-1)(-2y)}{(1-t^2-y^2)^2}$ no new points of discontinuity



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