Math 2153, Quiz #6, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the gradient of the function $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$ and plot the gradient field and at least 4 level curves.

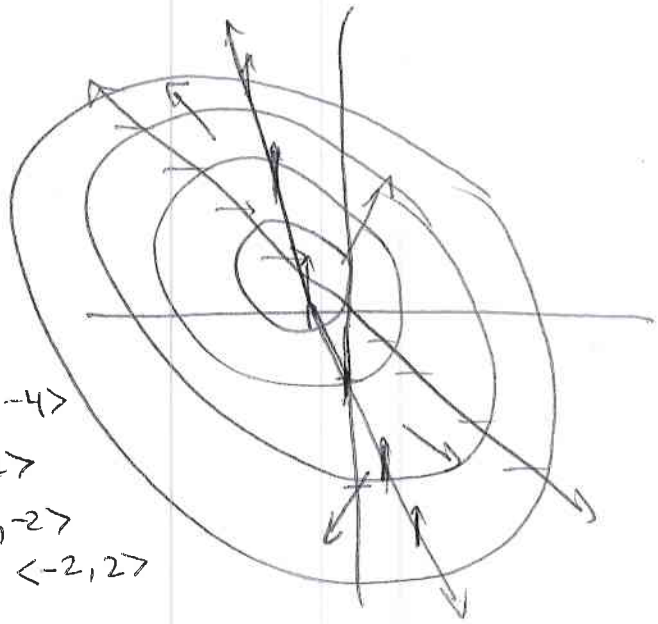
$\nabla f = \langle 4x+2y+2,\ 2x+2y \rangle$

$4x+2y+2=0$
$2x+y+1=0$
$y=-2x-1$

$2x+2y=0$
$y=-x$

(0,-2) plot points

$\nabla f(0,-2) = \langle 0-4+1,\ -4 \rangle = \langle -3,-4 \rangle$
$\nabla f(1,1) = \langle 4+2+2,\ 1+1 \rangle = \langle 8,2 \rangle$
$\nabla f(1,-3) = \langle 8-6+2,\ 4-6 \rangle = \langle 4,-2 \rangle$
$\nabla f(-3,4) = \langle -12+8+2,\ -6+8 \rangle = \langle -2,2 \rangle$



2. Find the extrema of the function $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$ using the second partials test and explain whether it agrees or disagrees with the graph above.

$\frac{\partial f}{\partial x} = 4x+2y+2=0$
$\frac{\partial f}{\partial y} = 2x+2y=0$

$\Rightarrow 2x + \underbrace{(2x+2y)}_{=0} + 2 = 0 \Rightarrow 2x+2=0 \Rightarrow x=-1$
$\Rightarrow y=1$

$D = 4(2) - 2^2 = 8-4 = 4 > 0$
max/min

$\frac{\partial^2 f}{\partial x^2} = 4$
$\frac{\partial^2 f}{\partial y^2} = 2$ $\Big\} > 0 \Rightarrow$ min at $(-1,1)$
$\frac{\partial^2 f}{\partial x \partial y} = 2$

agrees since gradient point to direction of increase and this graph increases away from critical point