Math 2415, Quiz #3, Summer 2015     Name KEY

**Instructions**: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. A mixing tank initially contains 140 gallons of brine which contains 25 pounds of salt in solution. A new brine containing 1.5 pounds of salt per gallon begins entering the tank at the rate of 2 gal/minute while the well-stirred mixture leaves the tank at 1 gal/min. Assuming the mixture is kept uniform, find the amount of salt in the tank at the end of an hour.

$Q(0) = 25$

$$\frac{dQ}{dt} = \frac{1.5\text{ lbs}}{\text{gal}} \cdot \frac{2\text{ gal}}{\text{min}} - \frac{Q\text{ lbs}}{140+t\text{ gal}} \cdot \frac{1\text{ gal}}{\text{min}}$$

$$\frac{dQ}{dt} = 3\text{ lbs/min} - \frac{Q}{140+t}\text{ lbs/min}$$

$$\frac{dQ}{dt} + \frac{1}{140+t}Q - 3 = 0 \qquad \mu = e^{\int \frac{1}{140+t}dt} = e^{\ln(140+t)} = 140+t$$

$$(140+t)\frac{dQ}{dt} + Q = 3(140+t)$$

$$\int [Q(140+t)]' = \int 3(140+t)$$

$$Q(140+t) = 3(140t + t^2/2) + C$$

$$Q = \frac{420t + \frac{3}{2}t^2 + C}{140+t}$$

$$25 = \frac{420t + \frac{3}{2}t^2 + C}{140+t} \qquad t=0$$

$$3500 + \cancel{25t} = \cancel{420t} + \cancel{\tfrac{3}{2}t^2} + C$$

$$3500 = C$$

$$Q(t) = \frac{420t + \frac{3}{2}t^2 + 3500}{140+t} \qquad Q(60) = 170.5\text{ lbs,}$$

2. A tank has pure water flowing into it at 20 L/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at the same rate. Initially, the tank contains 10 kg of salt in 100 L of water. Find the amount of salt in the tank at any time t.

$$\frac{dQ}{dt} = \frac{20L}{\text{min}} * 0 - \frac{Q}{100_5} \cdot \frac{20_1}{\text{min}}$$

$$Q(0) = 10$$
$$Q_0 = 10$$

$$\frac{dQ}{dt} = -\frac{Q}{5}$$

$$Q(t) = 10e^{-\frac{1}{5}t}$$

$$\int \frac{dQ}{Q} = \int -\frac{1}{5}dt$$

$$\ln Q = -\frac{1}{5}t + c$$

$$Q = e^{-\frac{1}{5}t + c} \Rightarrow Q(t) = Q_0 e^{-\frac{1}{5}t}$$