202 Homework #1 key

1a. $\left[\begin{array}{cc|c} 3 & 6 & -3 \\ 5 & 7 & 10 \end{array}\right]$ $\frac{1}{3}R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 5 & 7 & 10 \end{array}\right]$ $-5R_1 + R_2 \to R_2$ $\begin{matrix} -5 & -10 & 5 \end{matrix}$ $\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -3 & 15 \end{array}\right]$

$-\frac{1}{3}R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & -5 \end{array}\right]$ $-2R_2 + R_1 \to R_1$ $\begin{matrix} 0 & -2 & 10 \end{matrix}$ $\left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -5 \end{array}\right]$ $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

b. $\left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array}\right]$ $\frac{1}{2}R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array}\right]$ $-3R_1 + R_3 \to R_3$ $\begin{matrix} -3 & 0 & 9 & 12 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{array}\right]$ $-6R_2 + R_3 \to R_3$ $\begin{matrix} 0 & -6 & -12 & -18 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 5 & -10 \end{array}\right]$ $\frac{1}{5}R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$x_3 = -2$
$x_2 + 2x_3 = 3$
$x_2 + 2(-2) = 3$

$x_2 - 4 = 3 \quad x_2 = 7$
$x_1 - 3x_3 = -4$
$x_1 - 3(-2) = -4$

$x_1 + 6 = -4$
$x_1 = -10$
$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 7 \\ -2 \end{bmatrix}$

c. $\left[\begin{array}{cccc|c} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array}\right]$ $\frac{1}{2}R_1 \to R_1$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array}\right]$ $3R_1 + R_4 \to R_4$ $\begin{matrix} 3 & 0 & 0 & 6 & -15 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & 7 & -10 \end{array}\right]$ $\frac{1}{3}R_2 \to R_2$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & 7 & -10 \end{array}\right]$ $-2R_2 + R_4 \to R_4$ $\begin{matrix} 0 & -2 & -2 & 0 & 0 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & 7 & -10 \end{array}\right]$ $-R_3 + R_4 \to R_4$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 3 & -9 \end{array}\right]$ $\frac{1}{3}R_4 \to R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$

$x_4 = -3$
$x_3 + 4(-3) = -1$
$x_3 - 12 = -1$

$x_3 = 11$
$x_2 = -11$
$x_1 - 2(-3) = -5$

$x_1 + 6 = -5$
$x_1 = -11$
$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -11 \\ -11 \\ 11 \\ -3 \end{bmatrix}$

d. $\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -3 & -4 & 2 & 0 \end{array}\right]$ $\frac{1}{2}R_1 \to R_1$ $\left[\begin{array}{ccc|c} 1 & -5/2 & 4 & 0 \\ -3 & -4 & 2 & 0 \end{array}\right]$ $3R_1 + R_2 \to R_2$ $\begin{matrix} 3 & -15/2 & 12 & 0 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & -5/2 & 4 & 0 \\ 0 & -23/2 & 14 & 0 \end{array}\right]$

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1d. cont'd $-\frac{2}{23}R_2 \to R_2$ $\left[\begin{array}{ccc|c} 1 & -5/2 & 4 & 0 \\ 0 & 1 & -28/23 & 0 \end{array}\right]$ $\frac{5}{2}R_2 + R_1 \to R_1$

$0 \quad 5/2 \quad -70/23 \quad 0$

$\left[\begin{array}{ccc|c} 1 & 0 & 22/23 & 0 \\ 0 & 1 & -28/23 & 0 \end{array}\right]$ $x_2 + -\frac{28}{23}x_3 = 0$ $x_1 + \frac{22}{23}x_3 = 0$

$x_2 = \frac{28}{23}x_3$ $x_1 = -\frac{22}{23}x_3$

$x_3 = x_3$

$\vec{X} = \begin{bmatrix} -\frac{22}{23} \\ \frac{28}{23} \\ 1 \end{bmatrix} x_3$ or $t = 23x_3$ $= \begin{bmatrix} -22 \\ 28 \\ 23 \end{bmatrix} t$

e. $\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -4 & 9 & 0 \end{array}\right]$ $-2R_1 + R_3 \to R_3$

$-2 \quad 4 \quad 6 \quad 0$

$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 15 & 0 \end{array}\right]$ $\vec{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

f. $\left[\begin{array}{ccccc|c} 2 & 0 & 0 & -4 & 1 & 0 \\ 0 & 3 & 3 & 0 & -1 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ -3 & 2 & 3 & 1 & -2 & 0 \end{array}\right]$ $\frac{1}{2}R_1 \to R_1$, $\frac{1}{3}R_2 \to R_2$ $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1/2 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ -3 & 2 & 3 & 1 & -2 & 0 \end{array}\right]$ $3R_1 + R_4 \to R_4$

$3 \quad 0 \quad 0 \quad -6 \quad 3/2 \quad 0$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1/2 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ 0 & 2 & 3 & -5 & -1/2 & 0 \end{array}\right]$ $-2R_2 + R_4 \to R_4$

$0 \quad -2 \quad -2 \quad 0 \quad 2/3 \quad 0$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1/2 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ 0 & 0 & 1 & -5 & 1/6 & 0 \end{array}\right]$ $-R_3 + R_4 \to R_4$

$0 \quad 0 \quad -1 \quad -4 \quad -6 \quad 0$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1/2 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & -9 & -\frac{35}{6} & 0 \end{array}\right]$ $-\frac{1}{9}R_4 \to R_4$ $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 1/2 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & 1 & 35/54 & 0 \end{array}\right]$ $-4R_4 + R_3 \to R_3$, $2R_4 + R_1 \to R_1$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -113/54 & 0 \\ 0 & 1 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & 92/27 & 0 \\ 0 & 0 & 0 & 1 & 35/54 & 0 \end{array}\right]$ $-R_3 + R_2 \to R_2$ $\begin{bmatrix} 1 & 0 & 0 & 0 & -113/54 \\ 0 & 1 & 0 & 0 & -101/27 \\ 0 & 0 & 1 & 0 & 92/27 \\ 0 & 0 & 0 & 1 & 35/54 \end{bmatrix}$

$x_1 = 113/54\, x_5$
$x_2 = 101/27\, x_5$
$x_3 = -92/27\, x_5$
$x_4 = -35/54\, x_5$
$x_5 = x_5$

$\vec{X} = \begin{bmatrix} 113/54 \\ 101/27 \\ -92/27 \\ -35/54 \\ 1 \end{bmatrix} x_5 \Rightarrow$ $x_5 = 54t$ $\begin{bmatrix} 113 \\ 202 \\ -184 \\ -35 \\ 54 \end{bmatrix}$

2. $x_1 = 2x_3$
$x_2 = -x_3$
$x_3 = x_3$ skip

$\Rightarrow$ $x_1 - 2x_3 = 0$, $x_2 + x_3 = 0$ $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$ or any linear combination of these equations

i.e. $\left[\begin{array}{ccc|c} 2 & 0 & -4 & 0 \\ 0 & 3 & 3 & 0 \\ 5 & 1 & -9 & 0 \end{array}\right]$

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3\. a. $\begin{cases} 12a - 12b = 7 \\ 3a + 4b = 0 \end{cases}$

$$\left[\begin{array}{cc|c} 12 & -12 & 7 \\ 3 & 4 & 0 \end{array}\right] \xrightarrow{\frac{1}{12}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -1 & 7/12 \\ 3 & 4 & 0 \end{array}\right]$$

$$\begin{array}{c} -3R_1 + R_2 \to R_2 \\ -3 \quad 3 \quad -7/4 \end{array} \left[\begin{array}{cc|c} 1 & -1 & 7/12 \\ 0 & 7 & -7/4 \end{array}\right] \xrightarrow{1/7 R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -1 & 7/12 \\ 0 & 1 & -1/4 \end{array}\right] \; R_2 + R_1 \to R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{array}\right] \quad \begin{array}{l} a = 1/3 \\ b = -1/4 \end{array} \Rightarrow \begin{array}{l} x = 3 \\ y = -4 \end{array}$$

b. $\begin{cases} 2a + b - 2c = 5 \\ 3a - 4b = -1 \\ 2a + b + 3c = 0 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 5 \\ 3 & -4 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{array}\right] \xrightarrow{-R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 2 & 1 & -2 & 5 \\ 3 & -4 & 0 & -1 \\ 0 & 0 & 5 & -5 \end{array}\right]$$

$$\begin{array}{l} 1/2 R_1 \to R_1 \\ 1/5 R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1/2 & -1 & 5 \\ 3 & -4 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{c} -3R_1 + R_2 \to R_2 \\ -3 \quad -3/2 \quad 3 \quad -15 \end{array} \left[\begin{array}{ccc|c} 1 & 1/2 & -1 & 5 \\ 0 & -11/2 & 3 & -16 \\ 0 & 0 & 1 & -1 \end{array}\right] \; -\tfrac{2}{11} R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & -1 & 5 \\ 0 & 1 & -6/11 & -32/11 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{6/11 R_3 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1/2 & -1 & 5 \\ 0 & 1 & 0 & -38/11 \\ 0 & 0 & 1 & -1 \end{array}\right] \; \begin{array}{l} -1/2 R_2 + R_1 \to R_1 \\ R_3 + R_1 \to R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 63/11 \\ 0 & 1 & 0 & -38/11 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow \begin{array}{l} a = 63/11 \\ b = -38/11 \\ c = -1 \end{array} \Rightarrow \begin{array}{l} x = 11/63 \\ y = -11/38 \\ z = -1 \end{array}$$

4\. a. $\left[\begin{array}{cc|c} 4 & k & 6 \\ k & 1 & -3 \end{array}\right] \to *(-2)$ $-2R_2 \to R_2$

$$\left[\begin{array}{cc|c} 4 & k & 6 \\ -2k & -2 & 6 \end{array}\right] \xrightarrow{-R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 4 & k & 6 \\ -2k-4 & -2-k & 0 \end{array}\right]$$

$-2k - 4 = 0$ and $-2 - k = 0$

$-2k = 4$ $\qquad -2 = k$

$\frac{-2k}{-2} = \frac{4}{-2} \Rightarrow k = -2$

$\boxed{k = -2}$

b. $\left[\begin{array}{ccc|c} k & 2k & 3k & 4k \\ 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array}\right]$ $K \neq 0$, $1/k R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \end{array}\right] \; \begin{array}{l} -R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -4 \\ 0 & -5 & -5 & -7 \end{array}\right] \begin{array}{l} -R_2 \to R_2 \\ -1/5 R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & 1 & 7/5 \end{array}\right] \xrightarrow{-R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & -13/5 \end{array}\right]$$

$$-R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 13/5 \end{array}\right]$$

this will have exactly one solution as long as $k \neq 0$

4c. $\left[\begin{array}{ccc|c} 1 & 2 & k & 6 \\ 3 & 6 & 8 & 4 \end{array}\right] \begin{array}{l} -3R_1 + R_2 \to R_2 \\ -3 \quad -6 \quad -3k \quad -18 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & k & 6 \\ 0 & 0 & 8-3k & -12 \end{array}\right]$

no solution when $8-3k=0 \Rightarrow \frac{8}{3}=\frac{3k}{3} \Rightarrow \boxed{k=\frac{8}{3}}$

d. $\left[\begin{array}{cc|c} 1 & h & -5 \\ 2 & -8 & 6 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -2 \quad -2h \quad 10 \end{array} \left[\begin{array}{cc|c} 1 & h & -5 \\ 0 & -2h-8 & 16 \end{array}\right]$

to be consistent $-2h-8 \neq 0 \Rightarrow -2h \neq 8 \Rightarrow \boxed{h \neq -4}$

e. $\left[\begin{array}{cc|c} -4 & 12 & h \\ 2 & -6 & -3 \end{array}\right] \begin{array}{l} 2R_2 + R_1 \to R_2 \\ 4 \quad -12 \quad -6 \end{array} \left[\begin{array}{cc|c} -4 & 12 & h \\ 0 & 0 & h-6 \end{array}\right]$ to be consistent $h-6=0 \Rightarrow h=6$

f. $\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right] \; 2R_1 + R_3 \to R_3 \; \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & 2g+k \end{array}\right] \; R_2 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & 2g+h+k \end{array}\right]$ to be consistent $2g+h+k=0$

5a. true (as long as any scalar multiples of rows are not scaled to zero)
which is in the definition

b. false. it has six columns and 5 rows

c. s_i's are unique when this is true; but when a system has infinitely many solutions then the "numbers" s_i are parametric expressions

d. true

e. false; equivalent linear systems must have the same solution set

f. false, it applies to all matrices

g. true, when the solution set is infinite

h. false; this row says $x_4=0$ which is part of a consistent solution.

i. true

j. false; the pivot positions remain the same under row operations.

k. true

l. true

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8a. $a_0 + a_1 + a_2 + a_3 = 7$

$a_0 + 2a_1 + 4a_2 + 8a_3 = 17$

$a_0 + 3a_1 + 9a_2 + 27a_3 = 31$

$a_0 + 4a_1 + 16a_2 + 64a_4 = 65$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 1 & 2 & 4 & 8 & 17 \\ 1 & 3 & 9 & 27 & 31 \\ 1 & 4 & 16 & 64 & 65 \end{array}\right]$$

Cubic polynomial (4 points)

$$\vec{a} = \begin{bmatrix} -15 \\ 100/3 \\ -14 \\ 8/3 \end{bmatrix}$$

$p(x) = 8/3\,x^3 - 14x^2 + \frac{100}{3}x - 15$

b. $a_0 - 2a_1 + 4a_2 - 8a_3 + 16a_4 = 28$

$a_0 - a_1 + a_2 - a_3 + a_4 = 0$

$a_0 = -6$

$a_0 + a_1 + a_2 + a_3 + a_4 = -8$

$a_0 + 2a_1 + 4a_2 + 8a_3 + 16a_4 = 0$

quartic = 5 points

$$\left[\begin{array}{ccccc|c} 1 & -2 & 4 & -8 & 16 & 28 \\ 1 & -1 & 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -6 \\ 1 & 1 & 1 & 1 & 1 & -8 \\ 1 & 2 & 4 & 8 & 16 & 0 \end{array}\right] \quad \vec{a} = \begin{bmatrix} -6 \\ -3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$p(x) = x^4 - x^3 + x^2 - 3x - 6$

c. $(1-h)^2 + (3-k)^2 = r^2$ $\quad 1 - 2h + h^2 + 9 - 6k + k^2 = r^2 \Rightarrow h^2 + k^2 - 2h - 6k + 10 = r^2$

$(-2-h)^2 + (6-k)^2 = r^2$ $\quad 4 + 4h + h^2 + 36 - 12k + k^2 = r^2 \Rightarrow h^2 + k^2 + 4h - 12k + 40 = r^2$

$(4-h)^2 + (2-k)^2 = r^2$ $\quad 16 - 8h + h^2 + 4 - 4k + k^2 = r^2 \Rightarrow h^2 + k^2 - 8h - 4k + 20 = r^2$

$$\begin{array}{rl} & h^2 + k^2 - 2h - 6k + 10 = r^2 \\ -R_2 + R_3 \to R_3 & -12h + 8k - 20 = 0 \\ -R_1 + R_2 \to R_2 & 6h - 6k + 30 = 0 \end{array}$$

$$\begin{array}{rl} & h^2 + k^2 - 2h - 6k + 10 = r^2 \\ 2R_3 + R_2 \to R_2 & -12h + 8k - 20 = 0 \\ & 0 \quad -4k + 40 = 0 \end{array}$$

$\Rightarrow 4k = 40 \Rightarrow k = 10$

$-12h + 8(10) - 20 = 0$

$12h = 60 \Rightarrow h = 5$

$25 + 100 - 2(5) - 6(10) + 10 = r^2 \Rightarrow r = \sqrt{65}$

$125 - \cancel{10} - 60 + \cancel{10} = 65$

$(x-5)^2 + (y-10)^2 = 65$

d. $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

$\dfrac{(-5-h)^2}{a^2} + \dfrac{(1-k)^2}{b^2} = 1$ eq. 1

$\dfrac{(-3-h)^2}{a^2} + \dfrac{(2-k)^2}{b^2} = 1$ eq 2

$\dfrac{(-1-h)^2}{a^2} + \dfrac{(1-k)^2}{b^2} = 1$ eq 3

$\dfrac{(-3-h^2)}{a^2} + \dfrac{(0-k)^2}{b^2} = 1$ eq 4

eq 2 - eq 4

$\dfrac{(2-k)^2}{b^2} - \dfrac{k^2}{b^2} = 0 \quad * b^2$

$4 - 4k + \cancel{k^2} - \cancel{k^2} = 0 \qquad 4 = 4k \quad \boxed{k = 1}$

eq 1 - eq 3

$\dfrac{(-5-h)^2}{a^2} - \dfrac{(-1-h)^2}{a^2} = 0 \quad * a^2$

$25 + 10h + \cancel{h^2} - (1 + 2h + \cancel{h^2}) = 0$

$24 + 8h = 0 \qquad 8h = -24 \Rightarrow h = -3$

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$k=1,\ h=-3$

8d cont'd

eqn 4 $\quad \dfrac{\cancel{(-3+3)^2}^{\,0}}{\cancel{a^2}} + \dfrac{1}{b^2} = 1 \Rightarrow b=1$

eqn 1 $\quad \dfrac{(-5+3)^2}{a^2} + \dfrac{\cancel{(1-1)^2}^{\,0}}{\cancel{b^2}} = 1 \Rightarrow 4 = a^2 \Rightarrow a=2$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$$

9. $\begin{bmatrix} 1/1 & 1/2 & 1/3 \\ 2/1 & 2/2 & 2/3 \\ 3/1 & 3/2 & 3/3 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix}$

10. $\begin{bmatrix} 0 & -1 & -2 \\ 3 & 0 & -1 \\ 4 & 5 & 0 \end{bmatrix}$

$i > j$ row# > column# <lower half>

$i < j$ row# < column# <upper half>

11.a. $I_1(1+1+5+4) - 5I_2 = 20+30 \Rightarrow 11I_1 - 5I_2 = 50$

$I_2(1+5+3+1) - 5I_1 - 1I_3 = -30-10 \Rightarrow -5I_1 + 10I_2 - I_3 = -40$

$I_3(1+4+2+2) - I_2 - 2I_4 = 10+20 \Rightarrow -I_2 + 9I_3 - 2I_4 = 30$

$I_4(2+1+4+3) - 2I_3 = -20-10 \Rightarrow -2I_3 + 10I_4 = -30$

$$\left[\begin{array}{cccc|c} 11 & -5 & 0 & 0 & 50 \\ -5 & 10 & -1 & 0 & -40 \\ 0 & -1 & 9 & -2 & 30 \\ 0 & 0 & -2 & 10 & -30 \end{array}\right] \quad \vec{I} = \begin{bmatrix} 3.68 \\ -1.90 \\ 2.57 \\ -2.49 \end{bmatrix}$$

decimals ok.

b. $I_1(1+7+4) - 7I_2 - 4I_4 = 40 \Rightarrow 12I_1 - 7I_2 - 4I_4 = 40$

$I_2(7+2+6) - 7I_1 - 6I_3 = 30 \qquad -7I_1 + 15I_2 - 6I_3 = 30$

$I_3(6+3+5) - 6I_2 - 5I_4 = 20 \qquad -6I_2 + 14I_3 - 5I_4 = 20$

$I_4(4+4+5) - 5I_3 - 4I_1 = -10 \qquad -4I_1 - 5I_3 + 13I_4 = -10$

$$\left[\begin{array}{cccc|c} 12 & -7 & 0 & -4 & 40 \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{array}\right] \quad \vec{I} = \begin{bmatrix} 11.43 \\ 10.55 \\ 8.04 \\ 5.84 \end{bmatrix}$$

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11c.

$$I_1(3+1+4+1) - I_2 - I_4 - 4I_5 = 50 \Rightarrow 9I_1 - I_2 - I_4 - 4I_5 = 50$$

$$I_2(1+1+2+3) - I_1 - 2I_3 - 3I_5 = -30 \Rightarrow -I_1 + 7I_2 - 2I_3 - 3I_5 = -30$$

$$I_3(3+2+2+3) - 2I_2 - 3I_4 - 3I_5 = 20 \Rightarrow -2I_2 + 10I_3 - 3I_4 - 3I_5 = 20$$

$$I_4(3+1+1+2) - 3I_3 - I_1 - 2I_5 = -40 \Rightarrow -I_1 - 3I_3 + 7I_4 - 2I_5 = -40$$

$$I_5(3+3+2+4) - 4I_1 - 3I_2 - 3I_3 - 2I_4 = 0 \qquad -4I_1 - 3I_2 - 3I_3 - 2I_4 + 12I_5 = 0$$

$$\left[\begin{array}{ccccc|c} 9 & -1 & 0 & -1 & -4 & 50 \\ -1 & 7 & -2 & 0 & -3 & -30 \\ 0 & -2 & 10 & -3 & -3 & 20 \\ -1 & 0 & -3 & 7 & -2 & -40 \\ -4 & -3 & -3 & -2 & 12 & 0 \end{array}\right] \qquad \vec{I} = \begin{bmatrix} 4.69 \\ -3.57 \\ -.167 \\ -5.05 \\ .213 \end{bmatrix}$$

12 a. i. $\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ 0 & 4 \end{bmatrix}$ $A+B$

ii. $2\begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 6 \\ 12 & -12 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 14 & -12 \end{bmatrix}$ $2B-3C$

b. i. $4A = 4\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ -4 & 16 \end{bmatrix}$ ii. $-5H = -5\begin{bmatrix} 1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 10 \end{bmatrix}$

c. i $AB = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 27+1 & 9+0 \\ -9+4 & -3+0 \end{bmatrix} = \begin{bmatrix} 28 & 9 \\ -5 & -3 \end{bmatrix}$

ii. $BA = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 27-3 & 9+12 \\ 3+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 24 & 21 \\ 3 & 1 \end{bmatrix}$

iii. $DE = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}\begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} = \begin{bmatrix} 0+3-4 & -3-12+8 & 5+0-28 \\ 0+1+0 & 6-4+0 & -10+0+0 \\ 0-4-1 & -9+16+2 & 15+0-7 \end{bmatrix} = \begin{bmatrix} -1 & -7 & -23 \\ 1 & 2 & -10 \\ -5 & 9 & 8 \end{bmatrix}$

iv. $BF = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 9+6 & 27+12 & -18-3 & 0+15 \\ 1+0 & 3+0 & -2+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 15 & 39 & -21 & 15 \\ 1 & 3 & -2 & 0 \end{bmatrix}$

v. $GC = \begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 12+28 & -12-28 \\ 22+20 & -22-20 \\ 4-12 & -4+12 \end{bmatrix} = \begin{bmatrix} 40 & -40 \\ 42 & -42 \\ -8 & 8 \end{bmatrix}$

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12.c. vi. $DG = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6+33+8 & -7-15+12 \\ -12+11+0 & 14-5+0 \\ 18-44+2 & -21+20+3 \end{bmatrix} = \begin{bmatrix} 47 & -10 \\ -1 & 9 \\ -24 & 2 \end{bmatrix}$

vii. $EH = \begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$

not defined

The size of the rows in E does not match the size of the columns in H

viii. $BJ = \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 36-3 \\ 4+0 \end{bmatrix} = \begin{bmatrix} 33 \\ 4 \end{bmatrix}$

13. a.
$E+D = 600+A \Rightarrow -A+D+E = 600$
$A+300 = 700+B \Rightarrow A-B = 400$
$C+300 = D+200 \Rightarrow C-D = -100$
$B+500 = C+100 \Rightarrow B-C = -400$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 1 & 600 \\ 1 & -1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 1 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & 0 & -400 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1/2 & 250 \\ 0 & 1 & 0 & 0 & 1/2 & -150 \\ 0 & 0 & 1 & 0 & 1/2 & 250 \\ 0 & 0 & 0 & 1 & 1/2 & 350 \end{array}\right] \Rightarrow$

$A + 1/2E = 250$
$B + 1/2E = -150$
$C + 1/2E = 250$
$D + 1/2E = 350$
$E = E$

$\Rightarrow \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -2 \end{bmatrix}E + \begin{bmatrix} 250 \\ -150 \\ 250 \\ 350 \end{bmatrix}$

E free

there is not a single solution. it depends on the traffic on E.

b.
$400 + x_2 = x_4$
$300 = x^2 + x_3 + x_5$
$x_1 + x_3 = 600 + x_4$
$x_4 + x_5 = 100$

$\Rightarrow -x_1 + x_2 = -400$
$x_2 + x_3 + x_5 = 300$
$x_1 + x_3 - x_4 = 600$
$x_4 + x_5 = 100$

$\Rightarrow \left[\begin{array}{ccccc|c} -1 & 1 & 0 & 0 & 0 & -400 \\ 0 & 1 & 1 & 0 & 1 & 300 \\ 1 & 0 & 1 & -1 & 0 & 600 \\ 0 & 0 & 0 & 1 & 1 & 100 \end{array}\right]$

x_5 free, x_3 free

$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1 & 700 \\ 0 & 1 & 1 & 0 & 1 & 300 \\ 0 & 0 & 0 & 1 & 1 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$
$x_1 + x_3 + x_5 = 700$
$x_2 + x_3 + x_5 = 300$
$x_4 + x_5 = 100$
$x_3 = x_3$
$x_5 = x_5$

$= \vec{x} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}x_3 + \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}x_5 + \begin{bmatrix} 700 \\ 300 \\ 0 \\ 100 \\ 0 \end{bmatrix}$

there is not a single solution

c.
$x_1 + x_2 = 20$
$x_1 + 10 = x_5$
$20 + x_3 = x_4$
$10 + x_5 = x_4$
$x_2 + x_3 = 10 + 10$

$\Rightarrow x_1 + x_2 = 20$
$x_1 - x_5 = -10$
$x_3 - x_4 = -20$
$x_5 - x_4 = -10$
$x_2 + x_3 = 20$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 0 & 0 & -1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 0 & 20 \end{array}\right] \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 - x_5 = -10$
$x_2 + x_5 = 30$
$x_3 - x_5 = -10$
$x_4 - x_5 = 10$

x_5 free $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}x_5 + \begin{bmatrix} -10 \\ 30 \\ -10 \\ 10 \\ 0 \end{bmatrix}$

not a single solution