202 Homework #2 Key

1. $A = \begin{bmatrix} .95 & .05 & .15 \\ .01 & .88 & .20 \\ .04 & .07 & .65 \end{bmatrix}$ $\vec{x}_0 = \begin{bmatrix} 295 \\ 55 \\ 400 \end{bmatrix}$ Monday $\vec{x}_1 = A\vec{x}_0$ Tuesday $x_2 = A\vec{x}_1 = A^2\vec{x}_0$ Wednesday

$$X_2 = \begin{bmatrix} 373.765 \\ 174.148 \\ 202.087 \end{bmatrix} \approx \begin{bmatrix} 374 \\ 174 \\ 202 \end{bmatrix}$$ 374 cars at West, 174 at North and 202 at East

$\vec{x}_3$ = Thursday, $\vec{x}_4$ = Friday, $\vec{x}_5$ = Saturday, $\vec{x}_6$ = Sunday $\vec{x}_7$ = Monday

$$\vec{x}_7 = A^7 x_0 = \begin{bmatrix} 431.1623988 \\ 216.5070086 \\ 102.3305926 \end{bmatrix} \approx \begin{bmatrix} 431 \\ 217 \\ 102 \end{bmatrix}$$ 431 at West, 217 at North, 102 at East

2a. i. $A^T = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$

iii. $G^T = \begin{bmatrix} 6 & 11 & 2 \\ -7 & -5 & 3 \end{bmatrix}$

ii. $D^T = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & -4 \\ 4 & 0 & 1 \end{bmatrix}$

iv. $J^T = \begin{bmatrix} 4 & -1 \end{bmatrix}$

b. i. $A^{-1} = \frac{1}{12+1}\begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4/13 & -1/13 \\ 1/13 & 3/13 \end{bmatrix}$

ii. C^{-1} = does not exist (det is zero)

iii. $D^{-1} = \begin{bmatrix} 1/27 & -19/27 & -4/27 \\ 2/27 & -11/27 & -8/27 \\ 5/27 & 13/27 & 7/27 \end{bmatrix}$

iv. F^{-1} does not exist (not square)

c. $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & -1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1+4 & 3+8 & -2-2 & 0+10 \\ 3+8 & 9+16 & -6-4 & 0+20 \\ -2-2 & -6-4 & 4+1 & 0-5 \\ 0+10 & 0+20 & 0-5 & 0+25 \end{bmatrix} = \begin{bmatrix} 5 & 11 & -4 & 10 \\ 11 & 25 & -10 & 20 \\ -4 & -10 & 5 & -5 \\ 10 & 20 & -5 & 25 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 6 & 11 & 2 \\ -7 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 36+49 & 24+35 & 12-21 \\ 66+35 & 44+25 & 22-15 \\ 12-21 & 8-15 & 4+9 \end{bmatrix} = \begin{bmatrix} 85 & 59 & -9 \\ 101 & 69 & 7 \\ -9 & -7 & 13 \end{bmatrix}$$

F^TF is symmetric. GG^T is not.

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3. $f(x) = x^3 - 2x^2 + 5x - 10$ $\quad A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}$

$f(A) = A^3 - 2A^2 + 5A - 10I$

$= \begin{bmatrix} 4 & 6 & -12 \\ 3 & -11 & 21 \\ -15 & 9 & 25 \end{bmatrix}$ non-square matrices do not have defined powers since $m \times n$ cannot multiply an $m \times n$ matrix unless $m = n$.

4. $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $\quad A^2 - 4A - 5I = 0$ matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$ $\quad 2X + 3A = B \Rightarrow 2X = B - 3A$

$\Rightarrow X = \frac{1}{2}(B - 3A)$

$\frac{1}{2}\left\{ \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} - 3\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & 4 \end{bmatrix} \right\} = \frac{1}{2}\left\{ \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ -3 & 0 \\ -9 & -12 \end{bmatrix} \right\} = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ -1 & 0 \\ -13 & -13 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -1/2 & 0 \\ -13/2 & -13/2 \end{bmatrix}$

$= X$

6. a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

7a. true

b. false; the inverse of those operations will

c. false $(AB)^{-1}$ is $B^{-1}A^{-1}$

d. true

e. true

f. true as long as A is $n \times n$; false otherwise

8. a. not invertible since R_1 and R_2 are multiples of each other

b. not invertible since R_2 is all zeros

c. invertible since the matrix is triangular and the diagonal entries are all non-zero.

d. invertible since $\det(A) = 2$

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9a. $\begin{cases} 2x+3y=12 \\ 4x-y=10 \end{cases}$ $\quad \underbrace{\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 12 \\ 10 \end{bmatrix}}_{\vec{b}}$ $\quad A^{-1} = \frac{1}{-2-12}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1/14 & 3/14 \\ 4/14 & -2/14 \end{bmatrix}$

$$\begin{bmatrix} 1/14 & 3/14 \\ 4/14 & -2/14 \end{bmatrix}\begin{bmatrix} 10 \\ 12 \end{bmatrix} = \begin{bmatrix} 10/14 + 36/14 \\ 40/14 - 24/14 \end{bmatrix} = \begin{bmatrix} 46/14 \\ 16/14 \end{bmatrix} = \begin{bmatrix} 23/7 \\ 8/7 \end{bmatrix}$$

b. $\begin{cases} -x+5y=17 \\ 3x-4y=12 \end{cases}$ $\quad \begin{bmatrix} -1 & 5 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 12 \end{bmatrix}$ $\quad A^{-1} = \frac{1}{4-15}\begin{bmatrix} -4 & -5 \\ -3 & -1 \end{bmatrix} =$

$$\frac{1}{11}\begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 17 \\ 12 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 68+60 \\ 51+12 \end{bmatrix} = \begin{bmatrix} 108/11 \\ 63/11 \end{bmatrix}$$

c. $\begin{cases} 5x-y+2z=10 \\ 3x+2y-4z=16 \\ -4x-3y+z=7 \end{cases}$ $\quad A = \begin{bmatrix} 5 & -1 & 2 \\ 3 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} 2/13 & 1/13 & 0 \\ -1/5 & -1/5 & -2/5 \\ 1/65 & -19/65 & -1/5 \end{bmatrix}$

$$A^{-1}\begin{bmatrix} 10 \\ 16 \\ 7 \end{bmatrix} = \begin{bmatrix} 20/13 - 16/5 + 7/65 \\ 10/13 - 16/5 - 133/65 \\ 0 - 32/5 - 7/5 \end{bmatrix} = \begin{bmatrix} -101/65 \\ -291/65 \\ -39/5 \end{bmatrix}$$

d. $\begin{cases} x+y+z=9 \\ -x+2y-3z=14 \\ 3x-5y-2z=-18 \end{cases}$ $\quad A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 3 & -5 & -2 \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} 19/31 & 3/31 & 5/31 \\ 11/31 & 5/31 & -2/31 \\ 1/31 & -8/31 & -3/31 \end{bmatrix}$

$$A^{-1}\begin{bmatrix} 9 \\ 14 \\ -18 \end{bmatrix} = \begin{bmatrix} 171/31 + 42/31 - 90/31 \\ 99/31 + 70/31 + 36/31 \\ 9/31 - 112/31 + 54/31 \end{bmatrix} = \begin{bmatrix} 123/31 \\ 205/31 \\ -49/31 \end{bmatrix}$$

10. see last page

11. $P^2 = \begin{bmatrix} .4 & .15 & .15 \\ .28 & .53 & .17 \\ .32 & .32 & .68 \end{bmatrix}$ The proportion of voters from each party changes in two election cycles.

12. $\begin{bmatrix} 16 & 12 & 5 \\ 1 & 19 & 5 \\ 0 & 19 & 5 \\ 14 & 4 & 0 \\ 13 & 15 & 14 \\ 5 & 25 & 0 \end{bmatrix}$ message

encoded message:

43 6 9 −38 −45 −13 −42 −47 −14 44 16 10
49 9 12 −55 −65 −20

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13. message code
$$\begin{bmatrix} 112 & -140 & 83 \\ 19 & -25 & 13 \\ 72 & -76 & 61 \\ 95 & -118 & 71 \\ 20 & 21 & 38 \\ 35 & -23 & 36 \\ 42 & -48 & 32 \end{bmatrix} * A^{-1} = \begin{bmatrix} 8 & 1 & 22 \\ 5 & 0 & 1 \\ 0 & 7 & 18 \\ 5 & 1 & 20 \\ 0 & 23 & 5 \\ 5 & 11 & 5 \\ 14 & 4 & 0 \end{bmatrix}$$

HAVE_A_GREAT_WEEKEND

14. $X(I-D)=E$

$$D = \begin{bmatrix} .3 & .2 \\ .4 & .4 \end{bmatrix} \quad E = \begin{bmatrix} 10{,}000 \\ 20{,}000 \end{bmatrix} \quad I-D = \begin{bmatrix} .7 & -.2 \\ -.4 & .6 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} .7 & -.2 & 10{,}000 \\ -.4 & .6 & 20{,}000 \end{array}\right] \text{ rref} \Rightarrow \quad X = \begin{bmatrix} 29{,}411.76 \\ 52{,}941.18 \end{bmatrix}$$

10. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{bmatrix}$ $\quad \frac{1}{2}R_1 \to R_1 \quad E_1 = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{bmatrix}$ $\quad -10R_1 + R_3 \to R_3 \quad E_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 12 & 3 \end{bmatrix}$ $\quad 4R_2 + R_3 \to R_3 \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 7 \end{bmatrix} = U$ $\quad E_1 E_2 E_3 A = U \quad L = (E_1^{-1} E_2^{-1} E_3^{-1})$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & -4 & 1 \end{bmatrix}$$