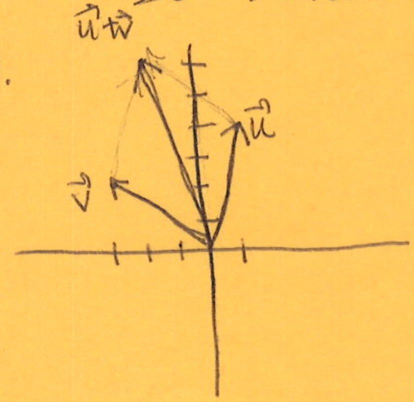
202 Homework #4 key

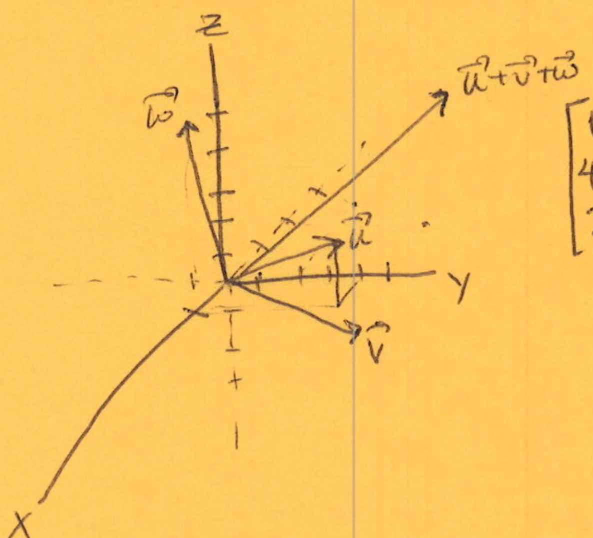
1.



$\vec{u}+\vec{v} = \begin{bmatrix}1\\4\end{bmatrix} + \begin{bmatrix}-3\\2\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix}$

if you imagine adding a copy of $\vec{v}$ to the end of $\vec{u}$ (and likewise a copy of $\vec{u}$ to the end of $\vec{v}$) we get a parallelogram and $\vec{u}+\vec{v}$ is the diagonal.

2.



$\begin{bmatrix}1\\4\\2\end{bmatrix} + \begin{bmatrix}-3\\2\\-3\end{bmatrix} + \begin{bmatrix}0\\-1\\5\end{bmatrix} = \begin{bmatrix}-2\\5\\4\end{bmatrix}$

3. a. $\vec{a}-\vec{b} = \begin{bmatrix}-1\\2\end{bmatrix} - \begin{bmatrix}3\\5\end{bmatrix} = \begin{bmatrix}-4\\-3\end{bmatrix}$

b. $3\vec{b}+2\vec{a} = 3\begin{bmatrix}3\\5\end{bmatrix} + 2\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}9\\15\end{bmatrix} + \begin{bmatrix}-2\\4\end{bmatrix} = \begin{bmatrix}7\\19\end{bmatrix}$

c. $\vec{c}+2\vec{d}-4\vec{e} = \begin{bmatrix}2\\7\\1\end{bmatrix} + 2\begin{bmatrix}1\\-1\\0\end{bmatrix} - 4\begin{bmatrix}4\\-5\\2\end{bmatrix} = \begin{bmatrix}2\\7\\1\end{bmatrix} + \begin{bmatrix}2\\-2\\0\end{bmatrix} + \begin{bmatrix}-16\\20\\-8\end{bmatrix} = \begin{bmatrix}-12\\25\\-7\end{bmatrix}$

4. a. false $f(t)=0$ for all t or its not the zero vector.

b. false it can be in higher dimensional spaces or not an "arrow".

c. false. this is one condition, but not the only one

d. true

e. true

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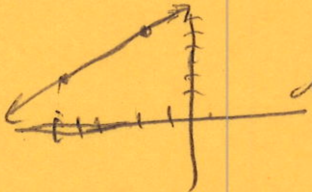
4f. false. R^2 is isomorphic to a subspace of R^3 but $\begin{bmatrix}a\\b\end{bmatrix}$ in R^2 is not in R^3.

g. true when $\vec{u}$ in H.

h. false  line does not go through origin

i. true

j. true

5. rref $\Rightarrow \left[\begin{array}{ccccc|c} 1&0&0&0&0&2\\ 0&1&0&0&0&1\\ 0&0&1&0&0&-2\\ 0&0&0&1&0&1\\ 0&0&0&0&1&-1 \end{array}\right] \Rightarrow 2\vec{v}_1+\vec{v}_2-2\vec{v}_3+\vec{v}_4-\vec{v}_5$

the solution is unique.

6. a. this is not a subspace since b^2 (the second component) is positive and so it will fail scalar multiplication.

$-1\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}-1\\-4\end{bmatrix}$ but $b^2\neq -4$ for any real b.

this condition (b^2) is equivalent to saying $y\geq 0$. //

b. This is a subspace. i) if $a=b=0, c=0$ then $\begin{bmatrix}0\\0\\0\end{bmatrix}$ in V. ii) if $\begin{bmatrix}a\\b\\c\end{bmatrix}$ in V, and $\begin{bmatrix}e\\f\\g\end{bmatrix}$ in V then $\begin{bmatrix}a+e\\b+f\\c+g\end{bmatrix}=\begin{bmatrix}(b+c)+(f+g)\\b+f\\c+g\end{bmatrix}=\begin{bmatrix}(b+f)+(c+g)\\b+f\\c+g\end{bmatrix}$ in V.

$=\begin{bmatrix}(b+c)\\b\\c\end{bmatrix}$ $\quad=\begin{bmatrix}f+g\\f\\g\end{bmatrix}$ since it follows the definition. iii) for

$k\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}ka\\kb\\kc\end{bmatrix}=\begin{bmatrix}k(b+c)\\kb\\kc\end{bmatrix}=\begin{bmatrix}kb+kc\\kb\\kc\end{bmatrix}$ in V. So this is a subspace. //

c. this is not a subspace since $\vec{0}$ not in set. if $a=b=0$ we get $\begin{bmatrix}0&2\\0&0\end{bmatrix}\neq\begin{bmatrix}0&0\\0&0\end{bmatrix}$. //

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6d. If you imagine the complex plane, $a+bi$ can be thought of as $\begin{bmatrix} a \\ b \end{bmatrix}$. this is the same as a vector in R^2, so yes, it is a vector space. we can show this is a subspace carefully. i) is zero in the space? yes. if $a=b=0$ then $0+0i=0$ ✓ ii) If I add two complex numbers is the result complex? yes, $(a+bi)+(c+di) = (a+c)+(b+d)i$ w/ $a+c$ and $b+d$ real. iii) can we scale & be in the space? yes, since k real means $k(a+bi) = (ka)+(kb)i$ w/ ka, kb real. //

6e. $p(t)$ divisible by $t-1$ means $p(t) = (t-1)q(t)$ where $q(t)$ is any polynomial. This is a subspace. if $q(t)=0$, then $(t-1)(0) = 0$. So 0 in the space. ii) if $s(t)$ & $p(t)$ in J, then $s(t)+p(t) = (t-1)r(t) + (t-1)q(t) = (t-1)(r(t)+q(t))$. Since $r(t)$, $q(t)$ are polynomials, so is $(r(t)+q(t))$, so $s(t)+p(t)$ in J. iii) if k is real then $kp(t) = k(t-1)q(t) = (t-1)(kq(t))$ but $kq(t)$ is a polynomial so $kp(t)$ in J. //

f. yes, this is a subspace. i) $f(x)=0$ then $f(-x)=0$ so 0 is an odd function since $f(-x)=f(x)=-f(x)$. ii) if f and g are both odd then $f(x)+g(x) = -f(-x)+(-g(-x)) = -(f(-x)+g(-x))$ and so $-(f(x)+g(x)) = f(-x)+g(-x)$. So the sum of two odd functions is odd. iii) and $kf(x)$ is odd, then $kf(-x) = k(-f(x)) = -kf(x)$. which is also odd. //

g. all $A^2 = A$ $(n \times n)$. i) the zero matrix is in the space since $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
ii) if A, B in W, the $A^2 = A$ and $B^2 = B$. $(A+B)^2 = A+B$?
$A^2+AB+BA+B^2 = A+B$? this is not true for all A, B in W since

bg cont'd I in W and if $A = I$, then $A^2 + AB + BA + B^2 =$

$(A^2 + B^2) + 2B \neq A + B$ except in the special case when $B = 0$.

This is not a subspace. //

h. exponential functions are typically defined a^x for $a > 0$. so

there is no zero in the space. Even if we allow a^x for $a = 0$,

$a^x + b^x$ is not exponential. //

i. The set of all singular matrices does contain the zero matrix

(since the zero matrix is singular). But $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are

singular but $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is nonsingular. //