

**Instructions:** Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Write the matrix that rotates the vector  $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$  by  $120^\circ$ .

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$$

$$\begin{aligned} \cos(120^\circ) &= -1/2 \\ \sin(120^\circ) &= \frac{\sqrt{3}}{2} \end{aligned} \quad \begin{array}{c} \sqrt{3} \\ 2 \\ 1 \end{array}$$

2. Write the matrix that transforms the vector  $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$  by a) reflecting around the x-axis, b) rotating around the y-axis by  $60^\circ$ , c) scales the z-axis by 2, d) projects onto the xz-plane.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E_1 \qquad E_2 \qquad E_3 \qquad E_4$

$$E_4 E_3 E_2 E_1 = \begin{bmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 0 & 0 \\ \sqrt{3}/2 & 0 & 1 \end{bmatrix}$$