MAT 202, Quiz #3, Summer 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Show why the equations $(A+B)(A-B) = A^2 - B^2$ is not valid for general matrices. Use an example to illustrate.

$(A+B)(A-B) = A^2 - AB + BA - B^2$

$-AB + BA$ only cancel if $AB = BA$

Consider $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 5 \\ 1 & -3 \end{bmatrix}$

$AB = \begin{bmatrix} 4 & -1 \\ 10 & 3 \end{bmatrix}$ $BA = \begin{bmatrix} 17 & 24 \\ -8 & -10 \end{bmatrix}$ so $AB \neq BA$

$(A+B)(A-B) = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -15 & -10 \\ 34 & 21 \end{bmatrix}$

but $A^2 - B^2 = \begin{bmatrix} -2 & 15 \\ 16 & 8 \end{bmatrix}$

2. Use row-reducing to find the inverse of $A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$. Find the inverse of the same matrix using the formula for 2x2 matrices. Do your results agree?

$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ -1 & 5 & 0 & 1 \end{array}\right]$ $R_1 \leftrightarrow R_2$ $\left[\begin{array}{cc|cc} -1 & 5 & 0 & 1 \\ 2 & 3 & 1 & 0 \end{array}\right]$ $-1R_1 \to R_1$

$\left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 2 & 3 & 1 & 0 \end{array}\right]$ $\begin{matrix} -2R_1 + R_2 \to R_2 \\ -2 \quad 10 \quad 0 \quad 2 \end{matrix}$ $\left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 0 & 13 & 1 & 2 \end{array}\right]$ $\frac{1}{13}R_2 \to R_2$

$\left[\begin{array}{cc|cc} 1 & -5 & 0 & -1 \\ 0 & 1 & 1/13 & 2/13 \end{array}\right]$ $\begin{matrix} 5R_2 + R_1 \to R_1 \\ 0 \quad 5 \quad 5/13 \quad 10/13 \end{matrix}$ $\left[\begin{array}{cc|cc} 1 & 0 & 5/13 & -3/13 \\ 0 & 1 & 1/13 & 2/13 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 5/13 & -3/13 \\ 1/13 & 2/13 \end{bmatrix}$

formula: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \frac{1}{10+3}\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5/13 & -3/13 \\ 1/13 & 2/13 \end{bmatrix}$

yes, they agree