

KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}$, describe the row operation that was performed, and write an elementary matrix E such that $EA = B$.

$$R_1 \leftrightarrow R_3 \quad E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

2. Is the matrix $A = \begin{bmatrix} .3 & .1 & .8 \\ .5 & .2 & .1 \\ .2 & .7 & .1 \end{bmatrix}$ a stochastic matrix? If it is, find the steady state for the system.

it is stochastic. all columns add to 1.

(raised A to a big power until columns are identical)
done when converts to fractions

$$A^{60} \approx \begin{bmatrix} 65/163 & 65/163 & 65/163 \\ 47/163 & 47/163 & 47/163 \\ 51/163 & 51/163 & 51/163 \end{bmatrix}$$

to do by hand:

$$(P-I) = \begin{bmatrix} -.7 & .1 & .8 \\ .5 & -.2 & .1 \\ .2 & .7 & -.9 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & -65/51 \\ 0 & 1 & -47/51 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} x_1 &= \frac{65}{51} x_3 \\ x_2 &= \frac{47}{51} x_3 \\ x_3 &= x_3 \end{aligned}$$

$$\approx \begin{bmatrix} 65 \\ 47 \\ 51 \end{bmatrix} t \div 163$$

$$= \begin{bmatrix} 65/163 \\ 47/163 \\ 51/163 \end{bmatrix}$$

$$65 + 47 + 51 = 163$$