

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Prove that if V is the set of all $M_{3,2}$ matrices, and $W = \left\{ \begin{bmatrix} a & b \\ a+b & 0 \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$, that W is a subspace of V .

1) if $a=b=c=0$ then $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ in W $\vec{0}$ (zero) exists in the set W ✓

2) $\begin{bmatrix} a & b \\ a+b & 0 \\ 0 & c \end{bmatrix} + \begin{bmatrix} d & e \\ d+e & 0 \\ 0 & f \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ (a+d)+(b+e) & 0 \\ 0 & c+f \end{bmatrix}$ in W ✓

3) $k \begin{bmatrix} a & b \\ a+b & 0 \\ 0 & c \end{bmatrix} = \begin{bmatrix} ka & kb \\ ka+kb & 0 \\ 0 & kc \end{bmatrix}$ in W ✓

W is a subspace of V

2. Prove that the set of all even, ^{continuous} functions is a subspace of $C(-\infty, \infty)$.

even functions: $f(x) = f(-x)$

- 1) is zero in the set? yes. if $f(x)=0$, $f(-x)=0$ also so 0 is an even function so 0 in set. ✓

- 2) is the sum of two even functions even? $f(x) = f(-x)$, $g(x) = g(-x)$
 $(f+g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f+g)(-x)$ so yes,
 the sum is in the set. ✓

- 3) is a scalar multiple of an even function also even? yes.
 $f(x) = f(-x) \rightarrow kf(x) = kf(-x)$ ✓ so it's in the set.

the set of even functions is a subspace of $C(-\infty, \infty)$