212 Homework #3 Key

1a. $y' - 2y = 3e^{t}$ $\qquad \mu = e^{\int -2\,dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 3e^{-t} \Rightarrow \int (e^{-2t}y)' = \int 3e^{-t}$

$e^{2t}\cdot e^{-2t}y = (-3e^{-t} + C)\cdot e^{2t}$

$y = -3e^{t} + Ce^{2t}$

b. $\dfrac{ty' - y = -t^2e^{-t}}{t}$ $\qquad t>0$

$y' - \frac{1}{t}y = te^{-t}$ $\qquad \mu = e^{\int -\frac{1}{t}dt} = e^{-\ln t} = e^{\ln(1/t)} = 1/t$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t}$

$\int (\frac{1}{t}y)' = \int e^{-t} \Rightarrow \left(\frac{1}{t}y = -e^{-t} + C\right)t$

$y = -te^{-t} + Ct$

c. $\dfrac{t^3y' + 4t^2y = e^{-t}}{t^3}$; $y(-1) = 0$ $\qquad t<0$

$y' + \frac{4}{t}y = t^{-3}e^{-t}$ $\qquad \mu = e^{\int \frac{4}{t}dt} = e^{4\ln t} = e^{\ln t^4} = t^4$

$t^4y' + 4t^3y = te^{-t} \Rightarrow \int (t^4y)' = \int te^{-t}\,dt$ $\qquad u = t \quad dv = e^{-t}$; $du = dt \quad v = -e^{-t}$; $+\int e^{-t}\,dt$

$t^4y = -te^{-t} - e^{-t} + C$

$y = \frac{-1}{t^3e^t} - \frac{1}{t^4e^t} + \frac{C}{t^4}$ $\qquad 0 = \frac{-1}{(-1)^3e^{-1}} - \frac{1}{(-1)^4e^{-1}} + \frac{C}{(-1)^4}$

$0 = \cancel{\frac{1}{e^{-1}}} - \cancel{\frac{1}{e^{-1}}} + \frac{C}{1} \Rightarrow C = 0$

$y = -\frac{1}{t^3e^t} - \frac{1}{t^4e^t}$

d. $\dfrac{ty' + 2y = \sin t}{t}$ $\qquad t>0$ $\qquad \mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$y' + \frac{2}{t}y = \frac{\sin t}{t}$

1d. cont'd

$t^2y' + 2ty = t\sin t \Rightarrow \int (t^2y)' = \int t\sin t$

$t^2y = \int t\sin t\,dt \quad u = t \quad dv = \sin t$
$\qquad du = dt \quad v = -\cos t$

$= -t\cos t + \int \cos t\,dt$

$t^2y = -t\cos t + \sin t + C$

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t}$$

e. $y' - 2y = e^{2t}, \; y(0) = 2 \qquad \mu = e^{\int -2dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 1 \Rightarrow (e^{-2t}y)' = \int 1\,dt$

$e^{-2t}y = t + C$

$y = te^{2t} + Ce^{2t} \qquad 2 = 0e^0 + Ce^0 \Rightarrow$
$\qquad 2 = C$

$y = te^{2t} + 2e^{2t}$

2. you should obtain the same solutions as in #1.

3. $S' = rS + k \qquad S(0) = 8000 \qquad r = .1$
$\qquad S(3) = 0$

$S' = .1S + k$

$S' - \frac{1}{10}S = k \qquad \mu = e^{\int -\frac{1}{10}dt} = e^{-\frac{t}{10}}$

$e^{-\frac{t}{10}}S' - \frac{1}{10}e^{-\frac{t}{10}}S = ke^{-\frac{t}{10}}$

$\int (e^{-\frac{t}{10}}S)' = \int ke^{-\frac{t}{10}}dt \Rightarrow Se^{-\frac{t}{10}} = -10ke^{-\frac{t}{10}} + C \quad) * e^{\frac{t}{10}}$

$S = -10k + Ce^{\frac{t}{10}} \qquad 8000 = -10k + C$
$\qquad 0 = -10k + Ce^{3/10}$

$8000 = -10k + C$
$0 = 10k - Ce^{3/10}$

$8000 = C(1 - e^{3/10}) \Rightarrow C = \frac{8000}{1 - e^{3/10}} \approx -22{,}866.37$

$\frac{8000 + 22{,}866.37}{-10} = -3086.64 = k$

3 cont'd

$S = -10(-3086.64) - 22{,}866.37\, e^{t/10}$

$S = 30{,}866.40 - 22{,}866.37\, e^{t/10}$

payment each year is \$3,086.64

total payments \$9259.92

loan $\underline{-8000.00}$

\$1259.92 in interest over 3 years.

4. $\dfrac{dy}{dt} = \dfrac{(.5 + \sin t)}{5} y \qquad y(0) = 1$ when doubled

$\int \dfrac{dy}{y} = \frac{1}{5}(.5 + \sin t)\,dt = \int \frac{1}{10} + \frac{1}{5}\sin t$

$\ln y = \frac{1}{10}t + -\frac{1}{5}\cos t + C$

$y = Ae^{(\frac{1}{10}t - \frac{1}{5}\cos t)} \qquad 1 = Ae^{(0 - \frac{1}{5}(1))} = Ae^{-1/5}$

$A = e^{1/5}$

$y = e^{(\frac{1}{10}t - \frac{1}{5}\cos t + 1/5)}$

$2 = e^{(\frac{1}{10}t - \frac{1}{5}\cos t + 1/5)} \Rightarrow \ln 2 = \frac{1}{10}t - \frac{1}{5}\cos t + 1/5 \Rightarrow$

$\ln 2 - 1/5 = \frac{1}{10}t - \frac{1}{5}\cos t$

doubles at $t \approx 2.96$

doubles (and stays doubled or more after $t \approx 6.73$

doubling rate should not depend on initial conditions

5. $\dfrac{dT}{dt} = k(T - 70^\circ) \qquad T(0) = 200 \qquad T(1) = 190$

$\int \dfrac{dT}{T-70} = \int k\,dt$

$\ln(T-70) = kt + C$

$T - 70 = Ae^{kt}$

$T(t) = 70 + Ae^{kt}$

$200 = 70 + Ae^{0}$

$130 = A$

$190 = 70 + 130e^{k(1)}$

$120 = 130e^{k}$

$\frac{120}{130} = e^{k}$

$\ln\left(\frac{12}{13}\right) = k \approx -.08$

$T(t) = 70 + 130e^{-.08t}$

212 Homework #3 key cont'd

5. cont'd

$$150 = 70 + 130e^{-.08t}$$

$$80 = 130e^{-.08t}$$

$$\frac{80}{130} = e^{-.08t} \Rightarrow \frac{\ln\left(\frac{8}{13}\right)}{-.08} = \cancel{-.08}\,t$$

$$t \approx 6.0656$$ a little more than six minutes later

6:04
m sec

6. $v(0)=0$ $= r'(0)=0$ $r(0) = -5000$ opens chute at $t=10$
change to $mv' = 12v$ ground $=0$

$$mv' = 32 - .75v$$ $\frac{180 \text{ lbs}}{32} = \frac{45}{8}$ slugs

$$\frac{dv}{dt} = \frac{8}{45}\left(32 - \frac{3}{4}v\right)$$

$$\frac{dv}{dt} = -\frac{2}{9}\left(v - \frac{128}{3}\right) \qquad \int \frac{dv}{v - \frac{128}{3}} = \int -\frac{2}{9}$$

$$\ln\left(v - \frac{128}{3}\right) = -\frac{2}{9}t + C \Rightarrow v - \frac{128}{3} = Ae^{-2/9t}$$

$$v = \frac{128}{3} + Ae^{-2/9t} \Rightarrow 0 = \frac{128}{3} + Ae^{0} \Rightarrow A = -\frac{128}{3}$$

$$v(t) = \frac{128}{3} - \frac{128}{3}e^{-2/9t} \qquad 0 \le t \le 10$$

$$v(10) = \frac{128}{3} - \frac{128}{3}e^{-\frac{20}{9}} \approx 38.04 \text{ ft/sec}$$ (a) $v_1(10) = v_2(0)$

$$\frac{dv}{dt} = \frac{8}{45}(32 - 12v) = \frac{8}{\cancel{45}_{9}}\cdot\cancel{12}^{4}\left(\frac{8}{3} - v\right) = \frac{32}{9}\left(\frac{8}{3} - v\right) = -\frac{32}{9}\left(v - \frac{8}{3}\right)$$

$$\int \frac{dv}{v - 8/3} = \int -\frac{32}{9}dt \Rightarrow \cancel{\ln}(v - 8/3) = Ae^{-32/9t}$$

$$v(t) = 8/3 + Ae^{-32/9t} \Rightarrow 38.04 = 8/3 + Ae^{-32/9(10)}$$

$$\frac{35.37}{e^{-\frac{320}{9}}} = 9.778 \times 10^{16}$$

$$v(t) = \begin{cases} \frac{128}{3} - \frac{128}{3}e^{-2/9t} & 0 \le t \le 10 \\ 8/3 + 9.778 \times 10^{16} e^{-32/9t} & t > 10 \end{cases}$$ (until the ground)

212 Homework #3 Key Cont'd

6 cont'd.

$$\int v(t)\,dt = \int \frac{128}{3} - \frac{128}{3} e^{-32/9t}\,dt = \frac{128}{3}t + \frac{\cancel{128}^{64}}{\cancel{3}} \cdot \frac{9^3}{\cancel{32}} e^{-32/9t} + C$$

$$\frac{128}{3}t + 192e^{-32/9t} + C$$

$$-5000 = \frac{128}{3}(0) + 192(1) + C \quad \Rightarrow C = -5192$$

$$r(t) = \frac{128}{3}t + 192e^{-32/9t} - 5192$$

$$\int v(t)\,dt = \int \frac{8}{3} + 9.778\times10^{16} e^{-32/9t}\,dt = \frac{8}{3}t + 9.778\times10^{16}\left(-\frac{9}{32}\right)e^{-32/9t} + C$$

$$\frac{8}{3}t - 2.75\times10^{16} e^{-32/9t} + C$$

$$r(10) = \frac{128}{3}(10) + 192e^{-\frac{20}{9}} - 5192 = -4744.53 \text{ ft}$$

ⓑ 255.47 feet

ⓒ 8/3 ft/sec

$$-4744.53 = \frac{8}{3}(10) - 2.75\times10^{16} e^{-\frac{320}{9}} + C$$

$$C = -4761.24$$

$$r(t) = \begin{cases} \frac{128}{3}t + 192e^{-32/9t} - 5192 & 0 \le t \le 10 \\ \frac{8}{3}t - 2.75\times10^{16} e^{-32/9t} - 4761.24 & t > 10 \end{cases}$$

$$0 = 8/3t - 2.75\times10^{16} e^{-32/9t} - 4761.24$$

$$4761.24 = 8/3t - 2.75\times10^{16} e^{-32/9t}$$

$t = 1785.46$ seconds hits ground

29.75 minutes

29:46

$$v(1785.46) = 8/3 + 9.778\times10^{16} e^{-32/9(1785.46)} \approx 8/3 \text{ ft/sec}$$ when hitting ground.

or 1.82 miles/hr

$A(0) = 30$

7. $\frac{dA}{dt} = \frac{4L}{min} \cdot \frac{1g}{L} - \frac{5L}{min} \cdot \frac{Ag}{200L - t}$

$\frac{dA}{dt} = 4 - \frac{5A}{200-t} \Rightarrow \frac{dA}{dt} + \frac{5A}{200-t} = 4$

$\mu = e^{\int \frac{5}{200-t} dt} = e^{-5\ln(200-t)} = \frac{1}{(200-t)^5}$

$\frac{1}{(200-t)^5} A' + \frac{5}{(200-t)^6} A = \frac{4}{(200-t)^5}$

$\int \left(\left(\frac{1}{(200-t)^5} A \right)' \right) = \int 4(200-t)^{-5} dt$

$\frac{1}{(200-t)^5} A = \frac{4}{-4}(-1)(200-t)^{-4} + C \quad) * (200-t)^5$

$A = (200-t) + C(200-t)^5$

$30 = 200 - 0 + C(200)^5$

$\frac{-170}{200^5} = -5.31 \times 10^{-10} = C$

$A(t) = 200 - t - 5.31 \times 10^{-1} (200-t)^5$

8a. $(t-3)y' + (\ln t)y = 2t, \; y(1) = 2$ linear

$y' + \frac{\ln t}{t-3} y = \frac{2t}{t-3}$ defined on $t > 0$, $t \neq 3$ $\quad (0,3) \cup (3,\infty)$ $\quad t=1$ on $(0,3)$

b. $y' = (1-t^2-y^2)^{1/2}$ non-linear

$1-t^2-y^2 \geq 0 \quad 1 \geq t^2+y^2$ inside unit circle (or on)

$f(t,y) = (1-t^2-y^2)^{1/2} \quad \frac{\partial f}{\partial y} = \frac{1}{2}(1-t^2-y^2)^{-1/2}(-2y) = \frac{-y}{\sqrt{1-t^2-y^2}}$

$1-t^2-y^2 \neq 0$ inside unit circle only

8c. $(4-t^2)y' + 2ty = 3t^2 \qquad y(-3)=1$

$y' + \frac{2t}{4-t^2}y = \frac{3t^2}{4-t^2} \qquad y \neq 2,-2 \qquad (-\infty,-2)\cup(-2,2)\cup(2,\infty)$

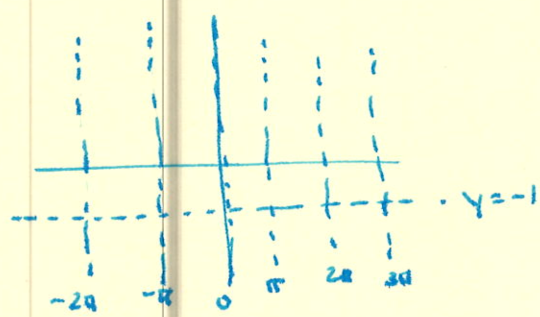
$y(-3)$ on $(-\infty,-2)$

d. $\frac{dy}{dt} = \frac{(\cot t)y}{1+y} \qquad t \neq k\pi \quad k \in \mathbb{Z}$

$y \neq -1$

$f(t,y) = (\cot t)\cdot\frac{y}{1+y}$

$\frac{\partial f}{\partial y} = \frac{1(1+y) - y(1)}{(1+y)^2} \qquad y \neq -1$



9a. $M = 3x^2 - 2xy + 2 \qquad N = 6y^2 - x^2 + 3$

$\frac{\partial M}{\partial y} = -2x \qquad \frac{\partial N}{\partial x} = -2x$ yes, it is exact

$\int 3x^2 - 2xy + 2\,dx = x^3 - x^2y + 2x + f(y)$

$\int 6y^2 - x^2 + 3\,dy = 2y^3 - x^2y + 3y + g(x)$

$\varphi(x): x^3 - x^2y + 2x + 2y^3 + 3y + K = 0$

b. $M = 9x^2 + y - 1 \qquad N = -4y + x$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 1$ yes, it is exact

$\int 9x^2 + y - 1\,dx = 3x^3 + xy - x + f(y)$

$\int -4y + x\,dy = -2y^2 + xy + g(x)$

$\varphi(x): 3x^3 + xy - x - 2y^2 = K \qquad 3(1)^3 + \cancel{(1)(0)} - (1) \cancel{- 2(0)^2} = K$

$3x^3 + xy - x - 2y^2 = 2 \qquad 3 - 1 = K \quad K = 2$

c. $M = ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x \qquad N = xe^{xy}\cos 2x - 3$

$\frac{\partial M}{\partial y} = e^{xy}\cos 2x + xye^{xy}\cos 2x - 2xe^{xy}\sin 2x \qquad \frac{\partial N}{\partial x} = e^{xy}\cos 2x + xye^{xy}\cos 2x - 2xe^{xy}\sin 2x$

yes, it is exact

$\int ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x\,dx = e^{xy}\cos 2x + x^2 + f(y)$

9c cont'd

$\int x e^{xy}\cos 2x - 3\, dy = e^{xy}\cos 2x - 3y + g(x)$

$\varphi(x,y):\ e^{xy}\cos 2x - 3y + x^2 = K$

10a. $y\,dx + (2x - ye^y)dy = 0 \qquad \mu(x,y) = y$

$y^2 dx + (2xy + y^2e^y)\,dy = 0$

$M \qquad N$

$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 2y$

$\int y^2 dx = xy^2 + f(y)$

$\int 2xy + y^2e^y\, dy = xy^2 + y^2e^y - 2ye^y + 2e^y + g(x)$

±	u	dv
+	y^2	e^y
−	$2y$	e^y
+	2	e^y
	0	e^y

$\varphi(x,y):\ xy^2 + y^2e^y - 2ye^y + 2e^y = K$

b. $dx + (\frac{x}{y} - \sin y)\,dy = 0 \qquad \mu = y$

$y\,dx + (x - y\sin y)\,dy = 0$

$M \qquad N$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 1$

$\int y\,dx = xy + f(y) \qquad \int x - y\sin y\, dy = xy - y\cos y + \sin y + g(x)$

±	u	dv
+	y	$\sin y$
−	1	$-\cos y$
+	0	$-\sin y$

$\varphi(x,y):\ xy - y\cos y + \sin y = K$

11. $t^2y' + 2ty - y^3 = 0 \qquad t > 0$

$t^2y' + 2ty = y^3 \qquad (1-n)y^{-n} = -2y^{-3} \qquad z = y^{-2}$

$z' = -2y^{-3}y'$

$-2t^2y^{-3}y' - 4ty^{-2} = -2$

$\frac{t^2z' - 4tz = -2}{t^2} \Rightarrow z' - \frac{4}{t}z = -2t^{-2} \qquad \mu = e^{\int -\frac{4}{t}dt} = e^{-4\ln t} = t^{-4}$

$t^{-4}z' - 4t^{-5}z = -2t^{-6}$

$\int (t^{-4}z)' = \int -2t^{-6}$

$\left(t^{-4}z = \frac{-2}{-5}t^{-5} + C\right)t^4$

$z = \frac{2}{5}\cdot\frac{1}{t} + Ct^4$

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11. cont'd $\frac{1}{y^2} = \frac{2}{5t} + Ct^4 = \frac{2+5Ct^5}{5t} \Rightarrow y^2 = \frac{5t}{2+5Ct^5}$

12a. $M = x-y \Rightarrow tx-ty = t(x-y)$
$N = x \Rightarrow tx = t(x)$ homogeneous of order 1

$y = vx \qquad y' = v'x+v$

$(x-y)dx = -x\,dy \Rightarrow \frac{dy}{dx} = \frac{x-y}{-x} \Rightarrow v'x+v = \frac{x-vx}{-x} = \frac{x(1-v)}{x(-1)}$

$v'x+v = v-1$
$\quad -v \quad -v$
$v'x = -1$

$\frac{dv}{dx}\cdot x = -1 \Rightarrow \int dv = \int \frac{-1}{x}dx$
$v = -\ln x + C$

$y = -x\ln x + Cx$

b. $M = x+3y \Rightarrow tx+3ty = t(x+3y)$ homogeneous of order 1
$N = 3x+y \Rightarrow 3tx+ty = t(3x+y)$

$y = vx \quad y' = v'x+v \qquad v'x+v = \frac{x+3vx}{3x+vx} = \frac{x(1+3v)}{x(3+v)}$

$v'x = \frac{1+3v}{3+v} - v\cdot\frac{3+v}{3+v} = \frac{1+3v-3v-v^2}{3+v}$

$v'x = \frac{1-v^2}{3+v} \Rightarrow \int dv \cdot \frac{3+v}{1-v^2} = \int \frac{1}{x}dx$

$\frac{A}{1-v} + \frac{B}{1+v} = A+Av+B-Bv = 3+v$

$A+B = 3$
$A-B = 1 \Rightarrow 2A = 4$
$A = 2 \Rightarrow B = 1$

$\int \frac{2}{1-v} + \frac{1}{1+v}\,dv = \int \frac{1}{x}dx \Rightarrow 2\ln(1-v) + \ln(1+v) = \ln x + C$

$v = \frac{y}{x}$

$\frac{(1-v)^2}{1+v} = \ln x + C$

$\Rightarrow \frac{\left(1-\frac{y}{x}\right)^2}{1+y/x} = \ln x + C$

12c. $y\,dx + (x\ln x - x\ln y - x)\,dy = 0$
$y\,dx + (x(\ln(x/y)) - x)\,dy = 0$

$M = y \Rightarrow t(y)$
$N = tx\cdot\ln\left(\frac{tx}{ty}\right) - tx = tx\cdot\ln\left(\frac{x}{y}\right) - tx$
$= t\left(x\ln\left(\frac{x}{y}\right) - x\right)$

12c. cont'd homogeneous of order 1 $\qquad y(1)=e$

$y=vx \qquad y'=v'x+v$

$\left[x\ln\left(\frac{x}{y}\right)-x\right]\frac{dy}{dx} = -y\cancel{dx}$

$y' = \frac{-y}{x(\ln(\frac{x}{y})-1)} \Rightarrow v'x+v = \frac{-vx}{x(\ln(\frac{x}{vx})-1)} = \frac{-v}{\ln(\frac{1}{v})-1} = \frac{+v}{+\ln v+1}$

$v'x+v = \frac{v}{\ln v+1} \qquad v'x = \frac{v}{\ln v+1} - v\frac{(\ln v+1)}{\ln v+1} = \frac{v-v\ln v-v}{\ln v+1}$

$v'x = \frac{-v\ln v}{\ln v+1} \Rightarrow \int dv\cdot\frac{(\ln v+1)}{+v\ln v} = \int -\frac{1}{x}\,dx$

$\ln[v\ln v] = -\ln x + C \qquad (v\ln v)' =$

$\ln\left[\frac{y}{x}\ln(y/x)\right] = -\ln x + C \qquad \ln v + v\cdot\frac{1}{v} =$

$\ln\left[\frac{e}{1}\cdot\ln\left(\frac{e}{1}\right)\right] = \ln[e(1)] = 1 = \cancel{-\ln(1)}^{0} + C \qquad \ln v+1$

$C=1$

$\ln\left[\frac{y}{x}\ln(y/x)\right] = -\ln x + 1 \Rightarrow +\ln\frac{e}{x}$

$\frac{y}{x}\ln(y/x) = \frac{e}{x}$

d. $(y^2+yx)\,dx - x^2\,dy = 0 \qquad M = y^2+yx \Rightarrow t^2y^2 + ty\cdot tx = t^2(y^2+yx)$

$N = x^2 \Rightarrow t^2x^2$

homogeneous of order 2

$(y^2+yx)\cancel{dx} = x^2\frac{dy}{dx} \qquad y=vx$

$\frac{y^2+yx}{x^2} = \frac{dy}{dx} \Rightarrow \qquad y' = v'x+v$

$\frac{v^2x^2+vx^2}{x^2} = v'x+v \Rightarrow \frac{\cancel{x^2}(v^2+v)}{\cancel{x^2}} - v \Rightarrow v^2 = v'x$

$\int\frac{dv}{v^2} = \int\frac{1}{x}dx \Rightarrow \frac{-1}{v} = \ln x + C \Rightarrow -\frac{x}{y} = \ln x + C$

212 Homework #3 key cont'd

12e. $y' = \frac{xy}{x^2-y^2}$ $M = xy \Rightarrow tx \cdot ty \Rightarrow t^2(xy)$

$N = x^2-y^2 \Rightarrow t^2x^2 - t^2y^2 = t^2(x^2-y^2)$

homogeneous of order 2

$y = vx \quad v'x + v = y'$

$v'x + v = \frac{vx^2}{x^2-v^2x^2} = \frac{v}{1-v^2}$ $v'x = \frac{v}{1-v^2} - \frac{v(1-v^2)}{1-v^2} = \frac{v - v + v^3}{1-v^2}$

$v'x = \frac{v^3}{1-v^2} \Rightarrow \int dv \cdot \frac{1-v^2}{v^3} = \int \frac{1}{x}\,dx$

$\int v^{-3} - v^{-1}\,dv = \int \frac{1}{x}\,dx = -\frac{1}{2}\cdot\frac{1}{v^2} - \ln v = \ln x + C$

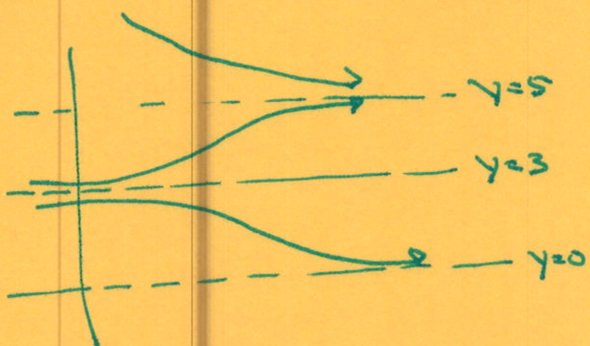
$-\frac{1}{2}\cdot\frac{x^2}{y^2} - \ln(y/x) = \ln x + C$

13. $y=5$ stable $\Rightarrow$ carrying capacity

$y=3$ unstable $\Rightarrow$ threshhold

$y=0$ semi-stable $\Rightarrow$ neither

logistic between $(0,3)$ & $(3,5)$



14. a. $\begin{cases} 2x+3y=12 \\ 4x-y=10 \end{cases}$

$\left[\begin{array}{cc|c} 2 & 3 & 12 \\ 4 & -1 & 10 \end{array}\right]$ $-2R_1 + R_2 \to R_2$
$-4 \quad -6 \quad -24$

$\left[\begin{array}{cc|c} 2 & 3 & 12 \\ 0 & -7 & -14 \end{array}\right]$ $\frac{1}{2}R_1 \to R_1$
$-\frac{1}{7}R_2 \to R_2$ $-\frac{2}{3}R_2 + R_1 \to R_1$

$\left[\begin{array}{cc|c} 1 & 3/2 & 12 \\ 0 & 1 & 2 \end{array}\right]$ $y=2$ $x+3=12$
$x=9$

$\left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 2 \end{array}\right]$

b. $\begin{cases} -x+5y=17 \\ 3x-4y=12 \end{cases}$ $\left[\begin{array}{cc|c} -1 & 5 & 17 \\ 3 & -4 & 12 \end{array}\right]$ $-R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & -5 & -17 \\ 3 & -4 & 12 \end{array}\right]$

$-3R_1 + R_2 \to R_2$
$-3 \quad 15 \quad 51$

$\left[\begin{array}{cc|c} 1 & -5 & -17 \\ 0 & 11 & 63 \end{array}\right]$ $\frac{1}{11}R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & -5 & -17 \\ 0 & 1 & \frac{63}{11} \end{array}\right]$ $5R_2 + R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & 0 & 128/11 \\ 0 & 1 & 63/11 \end{array}\right]$

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2 c.
$$\begin{cases} 5x - y + 2z = 10 \\ 3x + 2y - 4z = 16 \\ -4x - 3y + z = 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 5 & -1 & 2 & 10 \\ 3 & 2 & -4 & 16 \\ -4 & -3 & 1 & -7 \end{array}\right] \quad \tfrac{1}{5}R_1 \to R_1 \quad \left[\begin{array}{ccc|c} 1 & -1/5 & 2/5 & 2 \\ 0 & 13/5 & -26/5 & -10 \\ 0 & -19/5 & 13/5 & 15 \end{array}\right]$$

$-3R_1 + R_2 \to R_2$ $\quad$ $4R_1 + R_3 \to R_3$

$-3 \quad 3/5 \quad -6/5 \quad -6$ $\qquad$ $4 \quad -4/5 \quad 8/5 \quad 8$

$$\tfrac{5}{13}R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -1/5 & 2/5 & 2 \\ 0 & 1 & -2 & -50/13 \\ 0 & 0 & -5 & 5/13 \end{array}\right] \quad \tfrac{19}{5} R_2 + R_3 \to R_3$$

$0 \quad \frac{19}{5} \quad \frac{-38}{5} \quad \frac{-190}{13}$

$$-\tfrac{1}{5}R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & -1/5 & 2/5 & 2 \\ 0 & 1 & -2 & -50/13 \\ 0 & 0 & 1 & -1/13 \end{array}\right] \quad 2R_3 + R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & -1/5 & 2/5 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -1/13 \end{array}\right]$$

$0 \quad 0 \quad 2 \quad 2/13$

$-2/5 R_3 + R_1 \to R_1$

$1/5 R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 16/13 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -1/13 \end{array}\right]$$

d.
$$\begin{cases} x + y + z = 9 \\ -x + 2y + 3z = 14 \\ 3x - 5y - 2z = -18 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ -1 & 2 & 3 & 14 \\ 3 & -5 & -2 & -18 \end{array}\right] \quad \begin{array}{l} R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \\ -3 \quad -3 \quad -3 \quad -27 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 4 & 23 \\ 0 & -8 & -5 & -45 \end{array}\right]$$

$$1/3 R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 4/3 & 23/3 \\ 0 & -8 & -5 & -45 \end{array}\right] \quad 8R_2 + R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 4/3 & 23/3 \\ 0 & 0 & 17/3 & 49/3 \end{array}\right] \quad \tfrac{3}{17}R_3 \to R_3$$

$0 \quad 8 \quad \frac{32}{3} \quad \frac{184}{3}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 4/3 & 23/3 \\ 0 & 0 & 1 & 49/17 \end{array}\right] \quad -4/3 R_3 + R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 0 & 65/17 \\ 0 & 0 & 1 & 49/17 \end{array}\right] \quad \begin{array}{l} -R_3 + R_1 \to R_1 \\ -R_2 + R_1 \to R_1 \end{array}$$

$0 \quad 0 \quad -4/3 \quad -196/51$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 39/17 \\ 0 & 1 & 0 & 65/17 \\ 0 & 0 & 1 & 49/17 \end{array}\right]$$