212 Homework #5 Key

1. $A^{-1} = \frac{1}{2+4}\begin{bmatrix}1 & 1\\ -4 & 2\end{bmatrix} = \begin{bmatrix}1/6 & 1/6\\ -2/3 & 1/3\end{bmatrix}$

$B^{-1} = \frac{1}{-3i-(2-i)(1+i)}\begin{bmatrix}3 & -1-i\\ -2+i & i\end{bmatrix} = \frac{1}{-3-4i}\begin{bmatrix}3 & -1-i\\ -2+i & i\end{bmatrix} = \frac{1}{25}\begin{bmatrix}-9+12i & 7-i\\ 2-11i & -4-3i\end{bmatrix}$

$-3i-(2+1+2i-i)$

$-3i-(3+i)$

$-3i-3-i=-3-4i$

$C^{-1} = \frac{1}{5}\begin{bmatrix}-1 & 1 & -6\\ 6 & -1 & 16\\ -1 & 1 & -1\end{bmatrix}$

2. a. $\begin{cases}2x+3y=12\\ 4x-y=10\end{cases}$ $\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}^{-1} = \frac{1}{-2-12}\begin{bmatrix}-1 & -3\\ -4 & 2\end{bmatrix} = -\frac{1}{14}\begin{bmatrix}1 & -3\\ -4 & 2\end{bmatrix} = \begin{bmatrix}1/14 & 3/14\\ 4/17 & -2/14\end{bmatrix}$

$\frac{1}{14}\begin{bmatrix}1 & 3\\ 4 & -2\end{bmatrix}\begin{bmatrix}12\\ 10\end{bmatrix} = \frac{1}{14}\begin{bmatrix}12+30\\ 48-20\end{bmatrix} = \frac{1}{14}\begin{bmatrix}42\\ 28\end{bmatrix} = \begin{bmatrix}3\\ 2\end{bmatrix}$

b. $\begin{cases}-x+5y=17\\ 3x-4y=12\end{cases}$ $\begin{bmatrix}-1 & 5\\ 3 & -4\end{bmatrix}^{-1} = \frac{1}{4-15}\begin{bmatrix}-4 & -5\\ -3 & -1\end{bmatrix} = \frac{1}{11}\begin{bmatrix}4 & 5\\ 3 & 1\end{bmatrix}$

$\frac{1}{11}\begin{bmatrix}4 & 5\\ 3 & 1\end{bmatrix}\begin{bmatrix}17\\ 12\end{bmatrix} = \frac{1}{11}\begin{bmatrix}68+60\\ 51+12\end{bmatrix} = \frac{1}{11}\begin{bmatrix}128\\ 63\end{bmatrix} = \begin{bmatrix}128/11\\ 63/11\end{bmatrix}$

c. $\begin{cases}5x-y+2z=10\\ 3x+2y-4z=16\\ -4x-3y+z=7\end{cases}$ $\begin{bmatrix}5 & -1 & 2\\ 3 & 2 & -4\\ -4 & -3 & 1\end{bmatrix}^{-1} = \frac{1}{65}\begin{bmatrix}10 & 5 & 0\\ -13 & -13 & -26\\ 1 & -19 & -13\end{bmatrix}$

$\frac{1}{65}\begin{bmatrix}10 & 5 & 0\\ -13 & -13 & -26\\ 1 & -19 & -13\end{bmatrix}\begin{bmatrix}10\\ 16\\ 7\end{bmatrix} = \frac{1}{65}\begin{bmatrix}100+80+0\\ -130-208-182\\ 10-304-91\end{bmatrix} = \frac{1}{65}\begin{bmatrix}180\\ -520\\ -385\end{bmatrix}$ $\begin{bmatrix}36/13\\ -8\\ -77/13\end{bmatrix}$

d. $\begin{cases}x+y+z=9\\ -x+2y-3z=14\\ 3x-5y-2z=-18\end{cases}$ $\begin{bmatrix}1 & 1 & 1\\ -1 & 2 & -3\\ 3 & -5 & -2\end{bmatrix}^{-1} = \frac{1}{31}\begin{bmatrix}19 & 3 & 5\\ 11 & 5 & -2\\ 1 & -8 & -3\end{bmatrix}$

$\frac{1}{31}\begin{bmatrix}19 & 3 & 5\\ 11 & 5 & -2\\ 1 & -8 & -3\end{bmatrix}\begin{bmatrix}9\\ 14\\ -18\end{bmatrix} = \frac{1}{31}\begin{bmatrix}171+42-90\\ 99+70+36\\ 9-112+54\end{bmatrix} = \frac{1}{31}\begin{bmatrix}123\\ 205\\ -49\end{bmatrix}$

3. a. $\begin{vmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{vmatrix} = \cos^2 t + \sin^2 t = 1$

yes. this is a fundamental set

36.
$$\begin{vmatrix} t^2 & t^2\ln t & t^{-4} \\ 2t & 2t\ln t + t & -4t^{-5} \\ 2 & 2\ln t + 3 & 20t^{-6}\end{vmatrix} = t^2\left[(2t\ln t + t)(20t^{-6}) + 4t^{-5}(2\ln t + 3)\right] - t^2\ln t\left[2t\cdot 20t^{-6} + 8t^{-5}\right] +$$

$$+\ t^{-4}\left[2t(2\ln t + 3) - 2(2t\ln t + t)\right]$$

$$= t^2\left[40t^{-5}\ln t + 20t^{-5} + 8t^{-5}\ln t + 12t^{-5}\right] - t^2\ln t\left[40t^{-5} + 8t^{-5}\right]$$

$$+\ t^{-4}\left[4t\ln t + 6t - 4t\ln t - 2t\right]$$

$$= \cancel{40t^{-3}\ln t} + 20t^{-3} + \cancel{8t^{-3}\ln t} + 12t^{-3} - \cancel{40t^{-3}\ln t} - \cancel{8t^{-3}\ln t}$$

$$+ \cancel{4t^{-3}\ln t} + 6t^{-3} - \cancel{4t^{-3}\ln t} - 2t^{-3} =$$

$$t^{-3}\left[20 + 12 + 6 - 2\right] = 36t^{-3}$$

yes, a fundamental set

c.
$$\begin{vmatrix} x & xe^x \\ 1 & e^x + xe^x \end{vmatrix} = \begin{vmatrix} x & xe^x \\ 1 & (x+1)e^x \end{vmatrix} = x(x+1)e^x - xe^x = x^2e^x + \cancel{xe^x} - \cancel{xe^x}$$

$$= x^2e^x$$

yes, a fundamental set

d.
$$\begin{vmatrix} \sinh t & \cosh t & e^t \\ \cosh t & \sinh t & e^t \\ \sinh t & \cosh t & e^t \end{vmatrix} = e^t\underbrace{(\cosh^2 t - \sinh^2 t)}_{=1} - e^t\underbrace{(\sinh t\cosh t - \sinh t\cosh t)}_{=0}$$

$$+\ e^t\underbrace{(\sinh^2 t - \cosh^2 t)}_{=-1} = e^t(1 - 1) = 0$$

no, not a fundamental set

e.
$$\begin{vmatrix} e^t\sin t & e^t\cos t \\ e^t\sin t + e^t\cos t & e^t\cos t - e^t\sin t \end{vmatrix} = e^t\begin{vmatrix} \sin t & \cos t \\ \sin t + \cos t & \cos t - \sin t \end{vmatrix} =$$

$$e^t\left[\cancel{\sin t\cos t} - \sin^2 t - \cancel{\sin t\cos t} - \cos^2 t\right] = -e^t$$

yes, this is a fundamental set

Homework #5 key cont'd

4a. $\dfrac{ty'' + 3y = t}{t} \Rightarrow y'' + \frac{3}{t}y = 1$

coeff of $y' = p(t) = 0$

$W = e^{-\int 0\,dt} = c$

$\frac{3}{t}$ not defined at $t=0$

$y(1)$ defined at at least $(0, \infty)$

b. $\dfrac{t(t-4)y'' - 3ty' + 4y = 2}{t(t-4)}$ $y(3)$

not defined at $t=0, t=4$

defined on $(0, 4)$

$\Rightarrow y'' - \frac{3}{t-4}y' + \frac{4}{t(t-4)} = \frac{2}{t(t-4)}$

$W = ce^{\int \frac{3}{t-4}dt} = ce^{3\ln(t-4)} = ce^{\ln(t-4)^3} = c(t-4)^3$

c. $\dfrac{x^2(x^2-9)y'' - xy' + y = 0}{x^2(x^2-9)}$ $y(\frac{\pi}{2})$

not defined at $x = 0, \pm 3$

$y(\pi/2)$ defined on $(0, 3)$

$\Rightarrow y'' - \frac{1}{x(x^2-9)}y' + \frac{1}{x^2(x^2-9)}y = 0$

$W = ce^{\int \frac{1}{x(x^2-9)}dx} =$

$ce^{\int \frac{-1/9}{x} + \frac{1/18}{x-3} + \frac{1/18}{x+3}dx} =$

$ce^{-1/9 \ln x + 1/18 \ln(x-3) + \frac{1}{18}\ln(x+3)}$

$ce^{\ln x^{-1/9} + \ln(x-3)^{1/18} + \ln(x+3)^{1/18}}$

$ce^{\ln\left[\frac{(x-3)^{1/18}(x+3)^{1/18}}{x^{1/9}}\right]} =$

$c\sqrt[18]{\frac{(x-3)(x+3)}{x^2}}$

$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$

$Ax^2 - 9A + Bx^2 + 3Bx + Cx^2 - 3Cx = 1$

$A + B + C = 0$

$3B - 3C = 0 \Rightarrow B - C = 0 \Rightarrow B = C$

$-9A = 1 \Rightarrow A = -\frac{1}{9}$

$-\frac{1}{9} + 2B = 0 \quad B = 1/18 = C$

5a. $e^{1+2i} = e^1 e^{2i} = e(\cos 2 + i\sin 2) = e\cos 2 + ie\sin 2$

b. $2^{1-i} = e^{(\ln 2)(1-i)} = e^{\ln 2} \cdot e^{-\ln 2 i} = e^{\ln 2}(\cos(\ln 2) - i\sin(\ln 2)) =$

$2\cos(\ln 2) - 2i\sin(\ln 2)$

c. $e^{2-\pi/2 i} = e^2 e^{-\pi/2 i} = e^2(\cos\pi/2 - i\sin\pi/2) = e^2\cos\pi/2 - e^2 i\sin\pi/2$

6a. $1-i =$ $\|1-i\| = \sqrt{1^2+1^2} = \sqrt{2}$

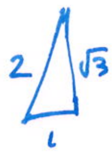
$\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$ $\cos\theta = \frac{1}{\sqrt{2}}$ $\sin\theta = \frac{-1}{\sqrt{2}}$ $\Rightarrow \tan^{-1}\left(\frac{-1}{1}\right) = -\pi/4 + \pi = 3\pi/4$

$= \sqrt{2}\, e^{3\pi/4 i}$

b. $1+2i$ $\|1+2i\| = \sqrt{1^2+2^2} = \sqrt{5}$ $\tan^{-1}\left(\frac{2}{1}\right) \approx 1.107$ rad or $63.43°$

$\sqrt{5}\left(\frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}}i\right) \approx \sqrt{5}\, e^{1.107 i}$

c. $\sqrt{3} - i$ $\|\sqrt{3} - i\| = \sqrt{3+1} = \sqrt{4} = 2$



$2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$ $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6 = 11\pi/6$

$= 2e^{11\pi/6 i}$ or $2e^{-\pi/6 i}$

d. $-2i$ $\|-2i\| = 2$ $\sin\theta = -1$, $\theta = 3\pi/2$

$2(0 - i)$

$2e^{3\pi/2 i}$

7a. $y = t^n$ $y' = nt^{n-1}$, $n(n-1)t^{n-2} = y''$

x^n nx^{n-1} $n(n-1)x^{n-2}$

$\cancel{x^2 \cdot x^{n-2}}\, n(n-1) + \cancel{x x^{n-1}}\, n + \cancel{x^n} = 0$

$n(n-1) + n + 1 = 0$

$n^2 - n + n + 1 = 0 \Rightarrow n^2 + 1 = 0$ $n = \pm i$ $x^i = e^{i\ln x} = \cos(\ln x) + i\sin(\ln x)$

$y(x) = c_1\cos(\ln x) + c_2\sin(\ln x)$

b. $t^2y'' + 5ty' + 13y = 0$

$n(n-1) + 5n + 13 = 0 \Rightarrow n^2 + 4n + 13 = 0$ $n = \frac{-4 \pm \sqrt{16-52}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i$

$y(t) = c_1 t^{-2}\cos(3\ln t) + c_2 t^{-2}\sin(3\ln t)$ $t^{-2+3i} = t^{-2}e^{\ln t \cdot 3i}$

c. $t^2y'' - ty' + 5y = 0$ $n(n-1) - n + 5 = 0 \Rightarrow n^2 - 2n + 5 = 0$

$n = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

$y(t) = c_1 t\cos(2\ln t) + c_2 t\sin(2\ln t)$

$2 \pm 4i$