MAT 212, Quiz #25, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use the Runge-Kutta method to estimate the solution to $y' = x + \sqrt{y}, y(0) = 1$ at $t = 1$ in two steps. $y_{n+1} = y_n + h\left(\frac{k_{n1}+2k_{n2}+2k_{n3}+k_{n4}}{6}\right)$,

$$k_{n1} = f(t_n, y_n), k_{n2} = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n1}\right),$$

$$k_{n3} = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n2}\right), k_{n4} = f(t_n + h, y_n + hk_{n3})$$

$x_0 =$

$t_0 = 0, \; y_0 = 1 \quad \Delta\cancel{x} = h = 1/2 \quad t_0 + 1/2 \cdot 1/2 = 1/4 \quad y_0 + 1/2 \cdot 1/2 \cdot 1 = 1 + 1/4 = 5/4 \quad y_0 + 1/4(1.368\ldots) = 1.342008497$

$k_{01} = 0 + \sqrt{1} = 1 \quad k_{02} = 1/4 + \sqrt{5/4} \approx 1.368033989$

$k_{03} = 1/4 + \sqrt{1.342\ldots} = 1.408450904 \quad t_0 + 1/2 = 1/2 \quad y_0 + 1/2(1.40845\ldots) = 1.704225452$

$k_{04} = (1/2 + \sqrt{1.70422\ldots}) = 1.805459862$

$y_1 = 1 + 1/2 \cdot \frac{1}{6}(1.368\ldots + 1.342\ldots \times 2 + 1.40845 \times 2 + 1.8054\ldots) = 1.722867721$

Step 1

Step 2

$t_1 = 1/2 \quad y_1 = 1.722\ldots \quad k_{11} = 1/2 + \sqrt{1.722\ldots} = 1.812580558 \quad t_1 + 1/4 = 3/4 \quad y_1 + 1/4 \cdot 1.812\ldots = 2.176012286$

$k_{12} = 3/4 + \sqrt{2.176\ldots} = 2.225131472 \quad y_1 + 1/4 \cdot 2.225\ldots = 2.279150589$

$k_{13} = 3/4 + \sqrt{2.279\ldots} = 2.259685593 \quad t_1 + 1/2 = 1 \quad y_1 + 1/2 \cdot 2.259685593 = 2.852129568$

$k_{14} = 1 + \sqrt{2.852\ldots} = 2.688824908$

$y_2 = 1.722867721 + 1/2 \cdot 1/6(1.8125\ldots + 2 * 2.2251\ldots + 2 * 2.25968\ldots + 2.6888\ldots) =$

2.845454354

$y(1) \approx 2.8455$