

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Classify each differential equation as i) ordinary or partial, ii) linear or non-linear, iii) state its order.

a. $x \frac{dy}{dx} + 3y = 2x^5$

first order, linear, ordinary

b. $u \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = xy \frac{\partial^2 u}{\partial x^2}$

2nd order, partial, nonlinear

c. $y''' + \cos y = 0$

3rd order, ordinary, nonlinear

d. $y' + 4ty = e^t$

first order, linear, ordinary

2. Verify that $y = xe^{-2x}$ is a solution to $y'' + 4y' + 4y = 0$.

$$y' = e^{-2x} - 2xe^{-2x}$$

$$y'' = 2e^{-2x} - 2e^{-2x} + 4xe^{-2x} = -4e^{-2x} + 4xe^{-2x}$$

$$-4e^{-2x} + 4xe^{-2x} + 4(e^{-2x} - 2xe^{-2x}) + 4(xe^{-2x}) =$$

$$\underbrace{-4e^{-2x} + 4xe^{-2x} + 4e^{-2x}}_{=0} - 8xe^{-2x} + 4xe^{-2x} = 0$$

