Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the determinant of each matrix.

   a. $\begin{bmatrix} 1 & 4 & 2 \\ 5 & 1 & -2 \\ -2 & 3 & 1 \end{bmatrix}$

   b. $\begin{bmatrix} e^{-2t} & e^{3t} \\ -2e^{-2t} & 3e^{3t} \end{bmatrix}$

$1 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - 4 \begin{vmatrix} 5 & -2 \\ -2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 5 & 1 \\ 2 & 3 \end{vmatrix}$

$= 1(1+6) - 4(5-4) + 2(15+2)$

$7 - 4 + 34 = \boxed{37}$

$e^{-2t} 3e^{3t} + 2e^{-2t} e^{3t}$

$= 3e^{t} + 2e^{t} = $

$\boxed{5e^{t}}$

2. Solve the ODE $y'' + 5y' + 6y = \sin t$ by the method of undetermined coefficients.

$r^2 + 5r + 6 = 0$

$(r+2)(r+3) = 0$

$r = -2, -3$

$y_g(t) = c_1 e^{-2t} + c_2 e^{-3t}$

$Y(t) = y_p(t) = A\sin t + B\cos t$

$Y'(t) = A\cos t - B\sin t$

$Y''(t) = -A\sin t - B\cos t$

$-A\sin t - B\cos t + 5A\cos t - 5B\sin t$

$\qquad + 6A\sin t + 6B\cos t = \sin t$

$-A - 5B + 6A = 1 \quad \Rightarrow \quad 5A - 5B = 1$

$-B + 5A + 6B = 0 \quad\quad \underline{5A + 5B = 0} \quad \Rightarrow A + B = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad A = -B$

$10A = 1$

$A = \frac{1}{10}$

$B = -\frac{1}{10}$

$\boxed{y(t) = c_1 e^{-2t} + c_2 e^{-3t} + \frac{1}{10}\sin t - \frac{1}{10}\cos t}$