MAT 202, Quiz #18, Summer 2017 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Determine if the matrices $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & -1 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 4 & 2 \\ 1 & 4 & -1 & 0 \\ 0 & 2 & 0 & 3 \end{bmatrix}$ can be orthogonally diagonalized. Why or why not?

A cannot be orthogonally diagonalizable because it is not symmetric

B is orthogonally diagonalizable because it is symmetric

2. The matrix $A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$ is orthogonally diagonalizable. Find the eigenvectors and demonstrate this result.

$(-1-\lambda)(-2-\lambda)-1=0$

$\lambda^2+3\lambda+2-1=0$

$\lambda^2+3\lambda+1=0$

$\lambda = \frac{-3 \pm \sqrt{9-4}}{2}$

$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$ real

$\vec{v}_1 \cdot \vec{v}_2 = (1+\sqrt{5})(1-\sqrt{5})+4 =$

$1-5+4=0$

they are orthogonal

$\begin{bmatrix} -1-(-\frac{3}{2}+\frac{\sqrt{5}}{2}) & 1 \\ 1 & -2-(-\frac{3}{2}+\frac{\sqrt{5}}{2}) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}-\frac{\sqrt{5}}{2} & 1 \\ 1 & -\frac{1}{2}-\frac{\sqrt{5}}{2} \end{bmatrix}$

$x_1 = (\frac{1}{2}+\frac{\sqrt{5}}{2})x_2$

$x_2 =$ x_2

$\vec{v}_1 = \begin{bmatrix} 1+\sqrt{5} \\ 2 \end{bmatrix}$ & $\begin{bmatrix} 1-\sqrt{5} \\ 2 \end{bmatrix} = \vec{v}_2$