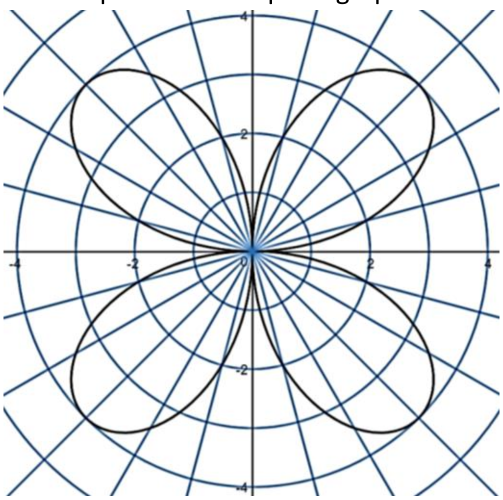
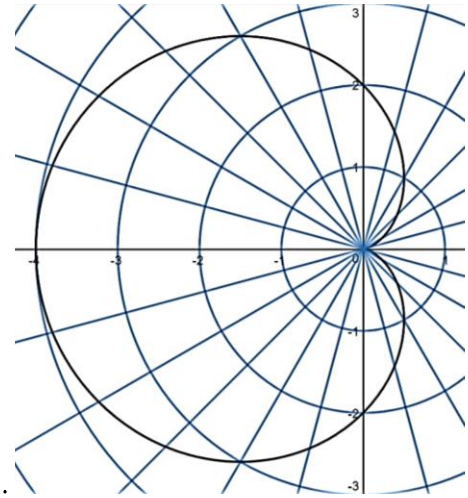
MTH 174, Practice Exam #4, Summer 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Use Taylor and power series to find a power series for expression.
   a. $f(x) = \frac{e^x - 1}{x}$
   b. $f(x) = \int \cos x^2 \, dx$
2. Sketch the parametric curve $x = t^3, y = 3\ln t$.
   a. Create a table of values (at least 5) for your graph.
   b. Label the orientation.
   c. Convert the equations back to rectangular coordinates.
   d. Find $\frac{dy}{dt}, \frac{dx}{dt}, \frac{dy}{dx}$.
   e. Find the equation of the tangent line at $t = 1$.
3. Sketch the parametric curve $x = \sec t\,, y = \tan t$.
   a. Create a table of values (at least 5) for your graph.
   b. Label the orientation.
   c. Convert the equations back to rectangular coordinates.
   d. Find $\frac{dy}{dt}, \frac{dx}{dt}, \frac{dy}{dx}$.
   e. Determine any points on the graph where the tangent lines are vertical or horizontal.
4. Find a parametric equation that passes through points $(1,4), and\ (5,-2)$.
5. Find the equation of the tangent line for the parametric curve given by $x = 3t - t^2, y = 2t^{3/2}$ at $t = \frac{1}{4}$.
6. Find the equation(s) of the tangent line(s) to the graph $r = 3\cos 2\theta$, passing through the pole.
7. Find the area of one petal of $r = 4\cos 3\theta$.
8. Find the area inside $r = 2\cos\theta$, and outside $r = 1$.
9. Sketch the vectors $\vec{u} = \langle 3,2 \rangle, \vec{v} = \langle -1,4 \rangle$ along with $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$. Explain how the graph illustrated the parallelogram rule.
10. Find the resulting force and direction of adding $||F_1|| = 75\ lbs., \theta_1 = 30°, ||F_2|| = 100\ lbs., \theta_2 = 45°, and\ ||F_3|| = 125\ lbs., \theta_3 = 120°$. Round answers to one decimal place.
11. Find the equation of the parabola with focus $(2,4)$, directrix $x = -4$. Sketch the graph.

12. Write the equation of the polar graphs below.

a. 

b. 

13. Use $\vec{u} = \langle 4,3 \rangle, \vec{v} = \langle 12,-5 \rangle$ to find the following.
   a. $\vec{u} + \vec{v}$

   b. $||\vec{u}||$

   c. Write $\vec{v}$ in polar form.

   d. Write a unit vector in the direction of $\vec{u}$

   e. Find $\vec{u} \cdot \vec{v}$

   f. Find the angle between $\vec{u}$ and $\vec{v}$. Report your answer is both radians (4 places) and degrees (1 place).

14. An ellipse has the endpoints of the major axis at $(7,9)$ and $(7,3)$, and one focus at $(7,8)$. Find the equation of the ellipse in standard form. (7 points)

15. Given the equation $r = \frac{8}{2+4\sin\theta}$, determine the type of conic this represents by finding the eccentricity of the graph. Then use technology to sketch the graph and confirm your results.

16. Graph the polar equation $r = 1 - 2\cos\theta$ on a polar graph. Clearly label at least 6 points and show work.

17. Graph the polar equation $r = \frac{1}{1-2\cos\theta}$ on a polar graph. Clearly label at least 6 points and show work.

18. A Hyperbola has a center at the origin, and vertices at $(0, \pm 4)$, foci at $(0, \pm 5)$. Write the equation of the hyperbola in standard form and sketch the graph.

19. An ellipse has major axis length 10, an eccentricity of $\frac{2}{5}$, center $(-2,3)$. Write the equation of the curve in standard form and sketch the graph.