MTH 265, Quiz #11, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the value of the work done in the vector field $\vec{F}(x,y,z) = (2x-y)\hat{\imath} + (z-y)\hat{\jmath} + (y-3z^2)\hat{k}$ on the path $\vec{r}(t) = t^2\hat{\imath} - t\hat{\jmath} + 3t\hat{k}$ on the interval [0,1]. If the field is conservative, use the fundamental theorem of line integrals.

$\int 2x - y\,dx = x^2 - xy$

$\int z - y\,dy = zy - \frac{1}{2}y^2$

field is not conservative no potential function

$$\int_C F \cdot dr =$$

$$\int_0^1 (2t^2+t)\,2t\,dt - (3t+t)\,dt + 3(-t-27t^2)\,dt$$

$$\int_0^1 4t^3 + 2t^2 - 4t - 3t - 81t^2\,dt = \int_0^1 4t^3 - 79t^2 - 7t\,dt =$$

$$t^4 - \frac{79}{3}t^3 - \frac{7}{2}t^2 \Big|_0^1 = 1 - \frac{79}{3} - \frac{7}{2} = -\frac{173}{6}$$

2. Use Green's Theorem to find the value of the line integral $\int_C (x-y)dx + (2x-3\cancel{x}y)dy$ around the boundary of the rectangle with the vertices (0,0), (3,0), (3,4), (0,4), clockwise.

$$\int_0^3 \int_0^4 2-(-1)\,dy\,dy = 12(3) = 36$$

3. Find the value of the surface integral $\int\int_S \vec{F}\cdot\vec{N}dS$ for the function $g(x,y,z)=x+y+z$ on the sphere $x^2+y^2+z^2=1$. (Use the back of the page.)

$N = \langle 2x, 2y, 2z \rangle$

$z = \sqrt{1-x^2-y^2}$

$F = \nabla g = \langle 1,1,1 \rangle$

$\iint_S 2x + 2y + 2\sqrt{1-x^2-y^2}\, dA = \int_0^{2\pi}\int_0^1 2r^2\cos\theta + 2r^2\sin\theta + 2r\sqrt{1-r^2}\, dr\, d\theta$

$\int_0^{2\pi}\int_0^1 2r^2\cos\theta + 2r^2\sin\theta + 2r\sqrt{1-r^2}\, dr\, d\theta$

$u = 1-r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$-\frac{1}{2}\int 2u^{1/2}\,du$

$\int_0^{2\pi} \frac{2}{3}r^3\cos\theta + \frac{2}{3}r^3\sin\theta - \frac{2}{3}(1-r^2)^{3/2}\Big|_0^1 d\theta$

$\int_0^{2\pi} \frac{2}{3}\cos\theta + \frac{2}{3}\sin\theta + \frac{2}{3}(1)^{3/2}\, d\theta$

$\frac{2}{3}\sin\theta - \frac{2}{3}\cos\theta + \frac{2}{3}\theta \Big|_0^{2\pi}$

$0 - \cancel{\frac{2}{3}(1)} + \frac{2}{3}(2\pi) - 0 + \cancel{\frac{2}{3}(1)} + 0$

$\frac{4\pi}{3}$