MTH 265, Quiz #14, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. An object is dropped into a gravity field with $\vec{a} = -5\hat{\jmath}$ ft/sec^2. It has initial velocity $\vec{v}(0) = 2\hat{\imath} + \hat{\jmath}$ and initial position $\vec{r}(0) = -\hat{\imath} + 300\hat{\jmath}$. Find the position function for the particle at time t>0. When and where does the particle hit the ground?

$$\int -5\hat{j}\,dt = C_1\hat{i} + (-5t+C_2)\hat{j} + C_3\hat{k} = \langle 2, 1, 0\rangle$$

$C_1 = 2 \quad C_3 = 0$

$-5t + C_2 = 1$

$t=0 \quad C_2 = 1$

$$v(t) = 2\hat{i} + (-5t+1)\hat{j} + 0\hat{k}$$

$$\int 2\hat{i} + (-5t+1)\hat{j} + 0\hat{k}\,dt = (2t+C_1)\hat{i} + (\tfrac{5}{2}t^2 + t + C_2)\hat{j} + C_3\hat{k}$$

$-\hat{i} \quad + 300\hat{j} \quad + 0k$

$C_1 = -1 \quad t=0 \quad C_2 = 300 \quad C_3 = 0$

position $r(t) = \langle 2t-1,\ 5/2t^2 + t + 300,\ 0\rangle$

2. Find the center of mass for the tetrahedron bounded by $x = 0, y = 0, z = 0, x + y + z = 1$ with density $\rho(x, y, z) = y$.

$z = -x-y+1 \qquad y = -x+1$

$$M = \int_0^1 \int_0^{-x+1} \int_0^{-x-y+1} y\,dz\,dy\,dx = \int_0^1 \int_0^1 -xy - y^2 + y\,dy\,dx = \int_0^1 -\tfrac{1}{2}xy^2 - \tfrac{1}{3}y^3 + \tfrac{1}{2}y^2\Big|_0$$

$$= \int_0^1 -\tfrac{1}{2}x(1-x)^2 - \tfrac{1}{3}(1-x)^3 + \tfrac{1}{2}(1-x)^2\,dx = \int_0^1 -\tfrac{1}{2}x(1-2x+x^2) - \tfrac{1}{3}(1-3x+3x^2-x^3) + \tfrac{1}{2}(1-2x\ldots\,dx$$

$$= \int_0^1 -\tfrac{1}{2}x + \cancel{x^2} - \tfrac{1}{2}x^3 - \tfrac{1}{3} + \cancel{x} - \cancel{x^2} + \tfrac{1}{3}x^3 + \tfrac{1}{2} - \cancel{x} + \tfrac{1}{2}x^2\,dx = \int_0^1 \tfrac{1}{6} - \tfrac{1}{2}x + \tfrac{1}{2}x^2 - \tfrac{1}{6}x^3\,dx =$$

$$\tfrac{1}{6}x - \tfrac{1}{4}x^2 + \tfrac{1}{6}x^3 - \tfrac{1}{24}x^4\Big|_0^1 = \tfrac{1}{6} - \tfrac{1}{4} + \tfrac{1}{6} - \tfrac{1}{24} = \tfrac{1}{24} \text{ mass}$$

$$M_{yz} = \int_0^1 \int_0^{-x+1} \int_0^{-x-y+1} xy\,dz\,dy\,dx = \cdots = \frac{1}{120} \qquad \bar{x} = \frac{\frac{1}{120}}{\frac{1}{24}} = \frac{1}{5}$$

$$M_{xz} = \int_0^1 \int_0^{-x+1} \int_0^{-x-y+1} y^2\,dz\,dy\,dx = \cdots = \frac{1}{60} \qquad \bar{y} = \frac{\frac{1}{60}}{\frac{1}{24}} = \frac{2}{5}$$

$$M_{xy} = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} zy\,dz\,dy\,dx = \cdots = \frac{1}{120} \qquad \bar{z} = \frac{\frac{1}{120}}{\frac{1}{24}} = \frac{1}{5}$$

Center of mass $\left(\frac{1}{5}, \frac{2}{5}, \frac{1}{5}\right)$