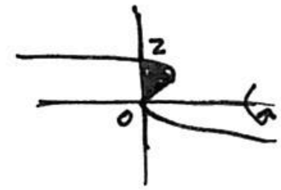
MTH 264, Quiz #2, Summer 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the volume of the solid of revolution for the region bounded by $x = 2y^2 - y^3, x = 0, y = 0$ rotated around the x-axis using method of cylindrical shells. Sketch the region.

$$2\pi\int_0^2 y(2y^2-y^3)\,dy = 2\pi\int_0^2 2y^3-y^4\,dy =$$

$$2\pi\left[\tfrac{2}{4}y^4 - \tfrac{1}{5}y^5\right|_0^2 = 2\pi\left[\tfrac{8}{5}\right] = \frac{16\pi}{5}$$



2. Find the surface area of the surface defined by $y = \frac{1}{x}$, on the interval [1,4], revolved around the y-axis.

$$y' = -\frac{1}{x^2}$$

$$2\pi\int_1^4 x\sqrt{1+\left(\tfrac{-1}{x^2}\right)^2}\,dx =$$

$$2\pi\int_1^4 x\sqrt{1+\tfrac{1}{x^4}}\,dx = 2\pi\int_1^4 x\sqrt{\tfrac{x^4+1}{x^4}}\,dx =$$

$$2\pi\int_1^4 \frac{x}{x^2}\sqrt{1+x^4}\,dx = 2\pi\int_1^4 \frac{1}{x}\sqrt{x^4+1}\,dx$$ integrate numerically

$$\approx 2\pi(7.71796) = 48.4934$$

3. Find an expression for the arc length of the function $f(x) = e^{-x}$ on the interval [-1,1]. Do not attempt to integrate by hand. (You can use your calculator to find the value.)

$$f'(x) = -e^{-x}$$

$$S = \int_{-1}^1 \sqrt{1+(-e^{-x})^2}\,dx = \int_{-1}^1 \sqrt{1+e^{-2x}}\,dx \approx 3.1962$$