MTH 264, Quiz #6, Summer 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Integrate.

a. $\int \frac{5x-12}{x^2-4}dx$

$(x-2)(x+2)$

$\frac{A}{x-2} + \frac{B}{x+2} = \frac{5x-12}{x^2-4} = \frac{A(x+2)+B(x-2)}{(x+2)(x-2)}$

$Ax+2A+Bx-2B = 5x-12$

$A+B=5$

$2A-2B=-12$

$2A+2B=10$

$2A-2B=-12$

$4A=-2$

$A=-\frac{1}{2}$

$-\frac{1}{2}+B=5 \rightarrow B=\frac{11}{2}$

$\int \frac{-\frac{1}{2}}{x-2} + \frac{\frac{11}{2}}{x+2}dx = \boxed{-\frac{1}{2}\ln|x-2| + \frac{11}{2}\ln|x+2| + C}$

b. $\int \frac{x^3}{\sqrt{4+x^2}}dx$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{4+4\tan^2\theta} = 2\sqrt{\sec^2\theta} = 2\sec\theta$

$\sqrt{4+x^2} =$

$\sec\theta = \frac{1}{2}\sqrt{4+x^2}$

$\int \frac{8\tan^3\theta \cdot \cancel{2}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{2}\cancel{\sec\theta}} = \int 8\tan\theta\sec\theta(\sec^2\theta-1)d\theta$

$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$\int 8(u^2-1)du = 8\left[\frac{1}{3}u^3 - u\right] + C$

$\frac{8}{3}\sec^3\theta - 8\sec\theta + C =$

$\boxed{\frac{4}{3}(4+x^2)^{3/2} - 4\sqrt{4+x^2} + C}$

c. $\int \frac{x^4-x^2+5x}{x^2-2x-1}dx =$

$$
\begin{array}{r}
x^2+2x+4 \\
x^2-2x-1\,\overline{)\,x^4+0x^3-x^2+5x+0} \\
-x^4+2x^3+x^2 \\
\hline
2x^3+0x^2+5x \\
-2x^3+4x^2+2x \\
\hline
4x^2+7x+0 \\
-4x^2+8x+4 \\
\hline
15x+4
\end{array}
$$

$x^2-2x+1-2$

$(x-1)^2-2$

$2(x-1)$ $\quad 2\left[\frac{15}{2}(x-1)+\frac{23}{2}\right]$

$\int x^2+2x+4+\frac{15x+4}{x^2-2x-1}dx$

$\int x^2+2x+4+\frac{15(x-1)}{(x-1)^2-2}+\frac{23}{(x-1)^2-2}dx$

$\frac{1}{3}x^3+x^2+4x+\frac{15}{2}\ln\left[x^2-2x-1\right]+\frac{23}{2\sqrt{2}}\ln\left|\frac{(x-1)-\sqrt{2}}{(x-1)+\sqrt{2}}\right|+C$

d. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}dx =$

$\text{arcsec}(x-1)+C$