

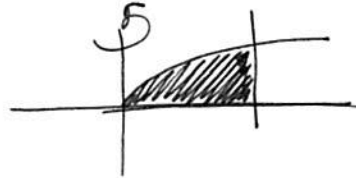
Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Sketch the graph of the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 1$ . Use the shell method to find the volume of the solid if this region is rotated around the  $y$ -axis.

$$x = y^2$$

$$2\pi \int_0^1 x(\sqrt{x}) dx = 2\pi \int_0^1 x^{3/2} dx =$$

$$2\pi \cdot \frac{2}{5} x^{5/2} \Big|_0^1 = 2\pi \cdot \frac{2}{5} (1) = \frac{4\pi}{5}$$



2. Set up the integral to find the arc length of the curve  $y = x^{3/2}$  on the interval  $[1, 8]$ . Use technology to evaluate it and report your answer to 4 decimal places.

$$y' = \frac{3}{2}x^{1/2} \quad (y')^2 = \frac{9}{4}x$$

$$S = \int_1^8 \sqrt{1 + \frac{9}{4}x} dx$$

$$u = 1 + \frac{9}{4}x$$

$$du = \frac{9}{4} dx$$

$$\frac{4}{9} du = dx$$

$$\frac{4}{9} \int u^{1/2} du = \frac{4}{9} u^{3/2} \cdot \frac{2}{3} \rightarrow$$

$$\frac{8}{27} \left[ \left(1 + \frac{9}{4}x\right)^{3/2} \Big|_1^8 \right] =$$

$$\frac{8}{27} \left[ \left(1 + \frac{9}{4} \cdot 8\right)^{3/2} - \left(1 + \frac{9}{4} \cdot 1\right)^{3/2} \right] =$$

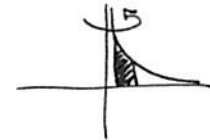
$$\frac{8}{27} \left[ 19^{3/2} - \left(\frac{13}{4}\right)^{3/2} \right]$$

$$\approx 22.80298\dots$$

3. Find the surface area if the curve  $y = \frac{1}{x}$  on the interval  $[\frac{1}{2}, 1]$  is rotated around the  $y$ -axis.

$$y' = -\frac{1}{x^2} \quad (y')^2 = \frac{1}{x^4}$$

$$2\pi \int_{1/2}^1 x \sqrt{1 + x^{-4}} dx = 2\pi \int_{1/2}^1 x \sqrt{1 + \frac{1}{x^4}} dx =$$



$$2\pi \int_{1/2}^1 x \sqrt{\frac{x^4 + 1}{x^4}} dx = 2\pi \int_{1/2}^1 x \frac{\sqrt{x^4 + 1}}{\sqrt{x^4}} dx = 2\pi \int_{1/2}^1 \frac{x}{x^2} \sqrt{x^4 + 1} dx$$

$$= 2\pi \int_{1/2}^1 \frac{\sqrt{x^4 + 1}}{x} dx \approx 0.798388\dots$$

4. A force of  $F = 20x - x^3$  stretches a nonlinear spring by  $x$  meters. What work is required to stretch the spring from  $x=1$  to  $x=3$  meters?

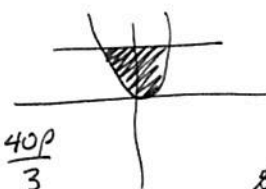
$$\int_1^3 20x - x^3 dx = 10x^2 - \frac{1}{4}x^4 \Big|_1^3 =$$

$$10(9) - \frac{1}{4}(81) - 10 + \frac{1}{4} =$$

$$90 - \frac{81}{4} - 10 + \frac{1}{4} = 80 - \frac{80}{4} = 80 - 20 = 60$$

5. Find the center of mass  $(\bar{x}, \bar{y})$  for the region bounded by  $y = \frac{5}{4}x^2$ ,  $y = 5$  assuming constant density.

$$M = \rho \int_{-2}^2 5 - \frac{5}{4}x^2 dx = \rho \left[ 5x - \frac{5}{12}x^3 \right]_{-2}^2 = \frac{40\rho}{3}$$



$$\begin{aligned} \frac{5}{4}x^2 &= 5 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

$$M_x = \frac{\rho}{2} \int_{-2}^2 5^2 - \left(\frac{5}{4}x^2\right)^2 dx = \frac{\rho}{2} \int_{-2}^2 25 - \frac{25}{16}x^4 dx =$$

$$\frac{\rho}{2} \left[ 25x - \frac{25}{16} \cdot \frac{1}{5}x^5 \right]_{-2}^2 = 40\rho$$

$$M_y = \rho \int_{-2}^2 x \left(5 - \frac{5}{4}x^2\right) dx = \rho \int_{-2}^2 5x - \frac{5}{4}x^3 dx = \rho \left[ \frac{5}{2}x^2 - \frac{5}{16}x^4 \right]_{-2}^2 = 0$$

$$\bar{x} = \frac{M_y}{M} = \frac{0}{\left(\frac{40\rho}{3}\right)} = 0$$

$$\bar{y} = \frac{M_x}{M} = \frac{40\rho}{\left(\frac{40\rho}{3}\right)} = \cancel{40\rho} \cdot \frac{3}{\cancel{40\rho}} = 3$$

Center is  $(0, 3)$