MTH 263, Quiz #6, Summer 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find $\frac{dy}{dx}$ using logarithmic differentiation for function $f(x) = x^{\ln x}$. $y = x^{\ln x}$

$\ln y = \ln(x^{\ln x}) \Rightarrow \ln y = (\ln x)(\ln x) \Rightarrow \ln y = (\ln x)^2$

$\frac{1}{y} y' = 2\ln x \cdot \frac{1}{x} \rightarrow y' = \frac{2y\ln x}{x} \rightarrow y' = \frac{2x^{\ln x}\ln x}{x}$

2. Find the indicated derivative of the given function.
   a. $f'(x), f(x) = \text{sech}\,(x)$

   $f'(x) = -\text{sech}\,x \tanh x$

   b. $f'(x), f(x) = \sinh^{-1} x$ $= \ln(x + \sqrt{x^2+1})$

   $f'(x) = \frac{1}{\sqrt{1+x^2}}$

   c. $f'(x), f(x) = \sinh(\cot x^2)$

   $f'(x) = \cosh(\cot x^2)(-\csc^2 x^2 \cdot 2x)$

3. Find the equation of the tangent line to the graph $f(x) = \tanh x$ at $x = \ln 2$.

$f'(x) = \text{sech}^2 x$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} = \frac{2 - \frac{1}{2}}{2 + \frac{1}{2}} = \frac{3/2}{5/2} = \frac{3}{5}$

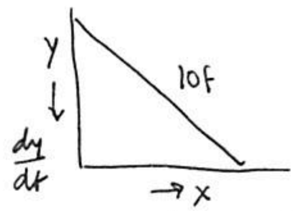
$(\ln 2, 3/5)$

$\text{Sech}\,x = \frac{4}{(e^x + e^{-x})^2} = \frac{4}{(e^{\ln 2} + e^{-\ln 2})^2} =$

$\frac{4}{(2 + 1/2)^2} = \frac{4}{(5/2)^2} = \frac{16}{25}$

$y - 3/5 = \frac{16}{25}(x - \ln 2)$

4. A 10 ft ladder is leaning against a wall and starts to slide away from the wall at a rate of $\frac{dx}{dt} = 0.4\frac{ft}{sec}$. How fast is the top of the ladder sliding down the wall when the foot of the ladder is three feet from the wall?



$\frac{dx}{dt} = 0.4\ \frac{ft}{sec}$

$X^2 + y^2 = 10^2$

$2xx' + 2yy' = 0$

$2x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$

$9 + y^2 = 100 \quad y^2 = 91 \quad y = \sqrt{91}$

$2(3)(0.4) + 2(\sqrt{91})\frac{dy}{dt} = 0$

$2.4 + 2\sqrt{91}\ \frac{dy}{dt} = 0$

$2\sqrt{91}\ \frac{dy}{dt} = -2.4$

$\frac{dy}{dt} = \frac{-2.4}{2\sqrt{91}} \approx -0.12579\ldots\ \frac{ft}{sec}$

5. Use differentials (linear approximations) to estimate the value of $\sqrt{35.95}$.

$y = \sqrt{x} \qquad X = 36,\ \Delta x = 0.05$ (negative)

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\Delta y \approx y'\Delta x = \frac{1}{2\sqrt{36}}(-0.05) = \frac{-0.05}{12} = -1/240$

$y(35.95) \approx y(36) + \Delta y = 6 - \frac{1}{240} \approx 5.9958\overline{3}$