

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integral $\int_1^3 \int_1^{5 \ln y} \frac{1}{xy} dy dx$.

$$\int_1^3 \frac{1}{x} \left[\int_1^{5 \ln y} \frac{1}{xy} dy \right] dx \quad \begin{array}{l} u = \ln y \\ du = \frac{1}{y} dy \end{array}$$

$$\int u du = \frac{1}{2} u^2 = \frac{1}{2} \ln^2 y$$

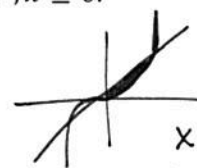
$$\begin{aligned} \int_1^3 \frac{1}{x} \left[\frac{1}{2} \ln^2 y \Big|_1^{5 \ln y} \right] dx &= \frac{1}{2} \int_1^3 \frac{1}{x} (\ln^2 5 - \ln^2 1) dx = \frac{\ln^2 5}{2} \int_1^3 \frac{1}{x} dx \\ &= \frac{\ln^2 5}{2} \ln x \Big|_1^3 = \frac{\ln^2 5}{2} (\ln 3 - \ln 1) = \frac{\ln^2 5 \cdot \ln 3}{2} \end{aligned}$$

2. Evaluate $\iint_R (x^2 + 2y) dA$ over the region R bounded by $y = x$, $y = x^3$, $x \geq 0$.

$$\int_0^1 \int_{x^3}^x (x^2 + 2y) dy dx =$$

$$\int_0^1 (x^2 y + y^2) \Big|_{x^3}^x dx = \int_0^1 (x^3 + x^2 - x^5 - x^6) dx$$

$$\left. \frac{1}{4} x^4 + \frac{1}{3} x^3 - \frac{1}{6} x^6 - \frac{1}{7} x^7 \right|_0^1 = \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7} = \frac{23}{84}$$



$$x = x^3$$

$$x = 0, 1, -1$$

$$x \geq 0 \Rightarrow x = 0, x = 1$$

3. Find the volume of the solid bounded by $y = 1 - x^2$, $y = x^2 - 1$, $x + y + z = 2$, $2x + 2y - z = 10$. Write a double integral and then evaluate it. Sketch the region in the plane.

$$\int_{-1}^1 \int_{x^2-1}^{1-x^2} \int_{2x+2y-10}^{2-x-y} dz dy dx = \int_{-1}^1 \int_{x^2-1}^{1-x^2} (2-x-y) - (2x+2y-10) dy dx$$

$$\int_{-1}^1 \int_{x^2-1}^{1-x^2} (12 - 3x - 3y) dy dx = \int_{-1}^1 (12y - 3xy - \frac{3}{2}y^2) \Big|_{x^2-1}^{1-x^2} dx =$$

$$\int_{-1}^1 (6x^3 - 2x^2 - 6x + 24) dx = \left. \frac{6}{4} x^4 - \frac{2}{3} x^3 - 3x^2 + 24x \right|_{-1}^1 = 32$$