

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate $\iint_R \frac{y^2}{x^2+y^2} dA$ where R is the region that lies between $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ in polar coordinates.

$$\int_0^{2\pi} \int_2^3 \frac{r^2 \sin^2 \theta}{r^2} r dr d\theta = \int_0^{2\pi} \int_2^3 r \sin^2 \theta dr d\theta =$$



$$\frac{1}{2} r^2 \Big|_2^3 \cdot \int_0^{2\pi} \sin^2 \theta d\theta = \frac{1}{2} (9-4) \int_0^{2\pi} \frac{1}{2} (1 - \cos 2\theta) d\theta =$$

$$\frac{5}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{2\pi} = \frac{5\pi}{2}$$

2. Find the volume of the solid bounded between $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$. Set up a double integral in polar coordinates and evaluate it.

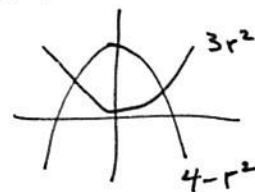
$$3x^2 + 3y^2 = 4 - x^2 - y^2$$

$$4x^2 + 4y^2 = 4$$

$$x^2 + y^2 = 1$$

$$3r^2$$

$$4-r^2$$



$$\int_0^{2\pi} \int_0^1 [(4-r^2) - 3r^2] r dr d\theta =$$



$$\int_0^{2\pi} \int_0^1 (4-4r^2) r dr d\theta = \int_0^{2\pi} \int_0^1 4r - 4r^3 dr d\theta =$$

$$\int_0^{2\pi} 2r^2 - r^4 \Big|_0^1 d\theta = \int_0^{2\pi} 2 - 1 d\theta = \int_0^{2\pi} d\theta = 2\pi$$